Entrepreneurship, Financial Frictions, and the Market for Firms^{*}

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Abstract

We study the relation between financial frictions and the trade of privately held firms. In the U.S. 20% of entrepreneurs purchased their business, however, this number has decreased in the last decades. In the cross section, younger, smaller, and high return to capital firms have the highest trading rates. To explain these findings, we propose a model of entrepreneurship and frictional trade of firms in which gains from trade arise from the presence of financial frictions. Our results suggest that the better allocation of capital due to the trade of firms accounts for 5-8% of entrepreneurial output and that easier access to credit can explain 40% of the decline in the trade of firms.

Keywords: entrepreneurship, financial frictions, firm ownership, misallocation, private firms, search frictions.

JEL classifications: E44, L20, G30.

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1 Introduction

Markets are the predominant allocation mechanism of modern economies. One important market, that allocates productive projects and available resources, is the market in which firms are bought and sold. In this paper, we argue that the role of this market is particularly relevant in economies in which financial constraints are a pervasive feature of entrepreneurial activity. In such environments, who owns the firms matters for allocations as credit constrained entrepreneurs will produce at a lower, and suboptimal, scale resulting in misallocation of capital and lower aggregate output. It might be the case, however, that financially constrained entrepreneurs might want to sell their firm to other less constrained parties.

We study this type of transactions in which gains from trading firms arise from the presence of financial frictions. Specifically, we ask: Is there evidence of this type of trades in the data? How important is this market, as an allocation mechanism, for the aggregate economy? We answer these questions by presenting new evidence about the trade of firms in the U.S. economy and by developing a model of entrepreneurship with a frictional market for firms with which we perform different quantitative experiments.

Our analysis starts with four new facts about the trade of *privately held firms*. We focus on private firms as these are the ones more likely to be affected by financial frictions, which dampen firms' access to external finance and insurance. Additionally, the study of private firms is important by itself as they account for over 70% of employment and around 50% of output in the U.S. economy (Asker, Farre-Mensa, and Ljungqvist, 2014; Dinlersoz et al., 2019). Throughout the analysis we focus on transactions where both the ownership and the management of the firm are traded. The aggregation of all these trades is what we call the *market for firms*.

For our empirical results we focus on *entrepreneurs*, defined as self-employed private business owners who actively manage their firm, and study how do they acquired their business. Using multiple data sources, we document that:

- 1. Around one out of five (or 20%) of entrepreneurs acquired their business by purchasing an existing firm, implying an annual trade rate of 2%.
- 2. The share of entrepreneurs that purchased their firm almost halved in the last 30 years. It went from around 25% in 1989 to roughly 15% in 2016 (see Figure 1).
- 3. More than 60% of firm buyers have never been entrepreneurs before purchasing their current business.
- 4. Younger, smaller, and high return to capital firms have the highest trading rates (see Figure 2).

Some comments about these findings are appropriate. First, the low trade rate we document indicates that private businesses are highly illiquid assets. Compared to housing, for example, Berger and Vavra (2015) report that 5% of houses are traded each year, more than two times the 2% annual trade rate that we find for private firms. Nonetheless, the trade of private firms is larger, in terms of volume, than the trade of specific intangibles assets such as patents. For example, Akcigit, Celik, and Greenwood (2016) document that 16% of the registered patents in the U.S. have been traded, smaller than the 20% we document for private businesses.

Regarding our second result, it is interesting that the fall in the trade of firms coincides with a period characterized by declining business dynamism and increasing concentration in the U.S. economy, suggesting a possible relation between these trends (see Akcigit and Ates (2021) for a recent review of that literature). As will be explained below, our model suggests an alternative explanation related to the economy credit conditions. Intuitively, if business owners have more access to external financing the gains from trading the ownership of their firms will be lower. This, in turn, will reduce the total number of trades in the economy.

Our third result, regarding the previous occupation of firm buyers, suggests that purchasing an existing firm is a relevant channel for entering into entrepreneurship which, to the best of our knowledge, has not been studied before. Our theoretical framework, besides capturing the illiquidity of private firms, will incorporate this novel feature about households' possible transitions into entrepreneurship through the market for firms.

Lastly, the results regarding firms' observable characteristics and their frequency of trade are highly informative about the underlying mechanisms behind the trade of firms. In this sense, any theory about the trade of private firms should be able to accommodate these relations. Both firms' age and size are associated with financial constraints (Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Further, firms' returns to capital are also informative about firms' access to external finance as credit constrained firms will have high returns but are not able to increase their investment. By introducing financial frictions as the micro foundation that generates gains from trading firms, our model can replicate the fact that younger, smaller, and high return to capital firms are the ones with the highest probabilities of trade.

Motivated by these findings, in the second part of the paper we develop a heterogeneous agent model with entrepreneurship and frictional trade of firms. Our model economy is populated by a continuum of households which can be firm owners or workers. Firm owners can trade or shutdown their firm, while workers can become firm owners by buying an existing firm or through some exogenous startup shock. Households are subject to uninsurable idiosyncratic risk. On the one hand, firm owners are exposed to the risk associated with the quality of their firm, which evolves stochastically. On the other hand, workers are subject to shocks to their labor efficiency. There is no aggregate uncertainty in this economy.

Firms are characterized by the quality of an entrepreneurial project which is indivisible, rival, and excludable. These entrepreneurial projects aim to capture firms' intangible assets.¹ Firms enable their owner to produce the final consumption good with a technology that uses capital, labor, and the quality of the firm. Besides the firms owned by a single household, which we call private firms, there is a second sector of production with a representative unconstrained public firm. Both sectors produce the same good which can be used for consumption or savings in a risk-free asset. There is also a financial intermediary that, each period, takes the savings from the households and rents capital to the firms.

Our empirical results about private firms being highly illiquid ask for a search-theoretic approach to model this market. Specifically, we model the market for firms through a decentralized market subject to *search frictions* and bilateral random matching. One interpretation of our setup is that agents can valuate only one firm at a time, which delays trade.² This setup will be suitable for our quantitative analysis as it gives us enough flexibility to match relevant features about the trade of firms documented in the empirical section of the paper.

Finally, we assume two types of *financial frictions*: incomplete markets which lead to uninsurable risk, and a collateral constraint limiting firm owners' use of external funds. In our baseline model financial frictions are the only motive behind the trade of firms.³ As a consequence, the typical sellers in our economy are firm owners with high quality firms but low wealth. These constrained owners are willing to sell their firms at relatively low prices as it would take them a long time, and high saving rates, to be able to grow out of their borrowing constraints. The typical buyers are wealthy households with low quality firms, or low labor efficiency.

We calibrate the parameters of the model to match several features of the U.S. economy for the year 2007. We target moments related to the role of entrepreneurs in the economy, the income and wealth distribution across households, the relative importance of the private business sector, and key characteristics of the market for firms. Regard-

¹Using data from business transactions, Bhandari and McGrattan (2021) document that when a firm is sold around 60% of its total value is accounted by intangibles. This evidence supports our modeling decision of characterizing firms by the value of their intangible assets.

²BizBuySell, an online marketplace for businesses, surveyed their clients about the major challenges they faced when purchasing a business. For more than 40% the major issue was "finding the right business", and for 23% was "valuating the firm". These responses are consistent with our modeling of the market for firms.

 $^{^{3}}$ As explained below, in our robustness section we extend the baseline model to allow for alternative motives behind the trade of firms.

ing the last group, specifically, we target the 2% annual trade rate and that 62% of the purchases are done by workers, capturing the importance of this market as a channel for entering entrepreneurship.

Our model has strong predictions about which firms have the highest probabilities of trade. If financial frictions are an important motive for the trade of firms credit constrained firms should be the ones more likely to be bought and sold. To validate our theory, we simulate data from our model and compare the implied relations between the trade rate and firms' observable characteristics with the data. In line with our empirical findings, our model predicts that younger, smaller, and high return to capital firms are the ones more likely to be traded.

It is important to note that these relations were *not* targeted in our baseline calibration strategy. Rather, they result from the key prediction that credit constrained firms are the ones more likely to be traded, and by the fact that these characteristics are strongly correlated with binding credit constraints in our model. Overall, we consider that these different pieces of evidence present a convincing case for financial frictions being an important motive behind the trade of privately held firms.

With our calibrated model we perform two counterfactual experiments to quantitatively assess the importance of this market. In our first experiment we take our baseline model and analyze a scenario in which the market for firms shuts down. We find that closing this market implies a fall in entrepreneurial output of 5.6%. This result is explained by the lower entrance into entrepreneurship together with the poorer allocation of productive projects and available resources when this market is absent.

The previous exercise indicates that the trade of firms is a way to alleviate the capital misallocation caused by financial frictions. To get a better sense of the gains in total factor productivity (TFP) that this market delivers, in our second experiment we consider an alternative economy with no trade of firms which we recalibrate to match the data. Then, we ask: what are the credit conditions that the no market economy requires such that it matches the TFP level of our baseline economy? We find that the no market economy requires looser credit conditions such that the debt to assets ratio of private firms increases by 7 percentage points (p.p.), from 0.37 to 0.44. This is a sizable amount as, for example, during the Great Recession this number fell by a total of 5 p.p.

Next, we study the relation between aggregate financial conditions and the trade of firms. In the empirical section of the paper, we documented that the share of entrepreneurs that purchased their firm fell from 25 to 15% in the last 30 years. An implication of our theory is that as credit conditions loosen, and current firm owners have more access to external finance, the gains from trading firms will be smaller, and hence there will be a reduction in the total number of trades.

Interestingly, during the period in which the fraction of firms purchased fell there was also an increase in private firms' leverage, which suggests an easier access to external financing. To quantitatively assess this relation, we fed into the model the change in leverage observed since the 1980s and analyze its implications for the trade of firms. Our model predicts a fall in the fraction of firms purchased of 4 p.p. This suggests that looser credit conditions can explain 40% of the 10 p.p. fall in the share of purchased firms observed during the last 30 years. Our results, however, do not rule out that the fall in the volume of traded firms could be related to other aggregate trends such as the decline in business dynamism in the U.S. economy.

In the last part of the paper, we redo our baseline exercise of shutting down the market for firms for three alternative economies. First, we consider an alternative parameterization that uses a narrower definition of entrepreneurs, which excludes firms with no employees, and features a higher annual trade rate. More trades imply an even more important role for the market for firms and hence shutting down this market will result on an even larger output fall. Second, we extend our baseline framework to include an alternative motive to trade firms that is orthogonal to financial frictions, such as firm owners' life cycle or retirement decisions. We find that even after accounting for trades unrelated to financial frictions the role of the market for firms as an allocation mechanism remains quantitatively unchanged. Finally, we consider an alternative model with no public firms, which implicitly assumes that also large corporate firms can be financially constrained. This model delivers a larger drop in aggregate output, however, stronger general equilibrium effects dampen the impact of shutting down this market.

Taking everything into account, we consider that our paper contributes to the literature in several ways. First, by documenting new facts about the trade of privately held firms in the U.S. economy. Second, by developing a novel model of entrepreneurship and frictional trade of firms that allow us to study the interaction between financial frictions and the market for firms. Third, by presenting new evidence that indicates that financial frictions are indeed an important motive for the trade of firms. Fourth, by quantifying the importance of this market and lastly by studying the fall in the trade of firms and its relation with the aggregate financial conditions.

1.1 Related Literature

Entrepreneurship and the Wealth Distribution. Our theoretical framework builds on the literature on heterogeneous agents models with entrepreneurship, such as Quadrini (2000) and Cagetti and De Nardi (2006). An important feature of these models is that they can match the observed income and wealth distribution through the combination of uninsurable income risk and stochastic returns to wealth coming from entrepreneurial activity.

From this literature, the closest to our paper is Peter (2020). This paper introduces the possibility that private firms go public through an Initial Public Offering (IPO). Then, it analyzes the implications of entrepreneurs' sources of external financing for wealth inequality. The main difference from our paper is that in Peter (2020) when entrepreneurs go public they maintain the management of the firm, regardless of how many shares they issue. In contrast, we study transactions in which both the ownership and the management of the firm are traded, which is a pervasive practice in the market for private firms.

Finance and Misallocation. Our paper also relates to the literature of financial frictions and misallocation as a source of low TFP (Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014). We contribute to this literature by highlighting the importance of the market for firms as a mechanism that alleviates the capital misallocation generated by financial frictions. In this sense, how this market functions across countries, due to institutional differences, the enforcement of contracts, or the rule of law, could have implications for the severity of financial frictions as a source of capital misallocation.

Trade of Ideas. From a modeling point of view, our paper is related to the literature that studies the trade of ideas, or patents, and its implications for economic growth, such as Silveira and Wright (2010) and Akcigit, Celik, and Greenwood (2016). Like our setup for the market for firms, both papers model the trade of ideas though a framework characterized by bilateral meetings subject to search frictions.

Technology Diffusion. Our paper also relates to Perla and Tonetti (2014) and Lucas and Moll (2014), among others. The technological diffusion process studied in those papers assume that firms' technology is non-rival, and hence technology adoption by other firms creates growth through imitation. On the contrary, in our model, private firms are rival goods. Hence, sellers need to be compensated upon a trade. Although there is a close relation with this literature, our paper is about the allocation of rival productive resources instead of technology diffusion.

Trade of Firms. Our paper relates to the literature that studies the trade of firms as an allocation mechanism of productive projects and available resources in the economy. Caselli and Gennaioli (2013) and Gaillard and Kankanamge (2020) focus on the trade of mature firms, which might arise from life-cycle considerations of firms' owners. David (2021) studies mergers and acquisitions of large firms, and Sevcik (2015) focuses on business groups. Notably, in Sevcik (2015), business groups are formed to curtail financial frictions as these groups can combine their resources to get more external financing. Lastly, Bhandari, Martellini, and McGrattan (2021) studies the trade of firms as a mechanism to accumulate capital when it is subject to indivisibilities. Distinctively, in our paper, we focus on the trade of financially constrained firms. Our theory is able to rationalize the fact that younger, smaller, and high returns to capital firms are the ones with the highest frequencies of trade.

The remainder of the paper is organized as follows: Section 2 presents our empirical results; Section 3 presents the model; Section 4 explains the properties of our model; Section 5 describes our parameterization strategy and presents evidence in favor of our theory of financial frictions being an important motive for the trade of firms; Section 6 presents our quantitative exercises; Section 7 discusses different extensions to our baseline model; and finally, Section 8 concludes.

2 Evidence on the Market for Firms

In this section we document relevant features about the market for firms. Our main results can be summarized as follows:

- 1. One out of five entrepreneurs purchased their business.
- 2. The share of entrepreneurs that purchased their firm has decreased in the last 30 years.
- 3. At least 60% of firm buyers have never been entrepreneurs.
- 4. Younger, smaller, and high returns to capital firms have the highest trading frequencies.

2.1 Data

We use four different surveys related to private firms, their characteristics, and the characteristics of their owners.⁴

SBO. Our main data source is the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS). This survey provides comprehensive information about businesses and business owners. In particular, about how do they acquired their business. The PUMS sample is representative of all non-farm private businesses in the U.S. and is available for the year 2007.

SCF. The second dataset we use is the Survey of Consumer Finances (SCF). We use the nine waves available between 1989 to 2016. The SCF includes detailed information about households' income and balance sheets, information that we will use to discipline the income and wealth distribution of our quantitative model. Additionally, this survey asks the business owners in the sample how do they acquired their firm. The information in the different waves of the SCF allows us to study how the ownership of firms have

⁴See Appendix A.1 for further details about these surveys and our sample selection criteria.

evolved over time.

ASE. Third, to get complementary information, we use the Annual Survey of Entrepreneurs (ASE). The ASE is available on a yearly basis from 2014 to 2016. As the SBO, this survey is representative of all non-farm private businesses in the U.S.

KFS. Lastly, we use data the Kauffman Firm Survey (KFS). This is an eight-year panel of businesses that started operations in 2004 and were followed through the year 2011. Compared to the previous datasets, the KFS contains more detailed information, particularly, about the balance sheet of firms.

2.2 Entrepreneurs

In our analysis we focus on *entrepreneurs* as the unit of observation. We follow Cagetti and De Nardi (2006) in defining entrepreneurs as the individuals who: (i) are self-employed, (ii) own a business, and (iii) have an active management role in it. Besides our baseline definition, we also look at entrepreneurs which, in addition to (i)-(iii), have (iv) a positive number of employees. This additional condition aims to exclude cases of self-employed individuals which businesses may not be transferable.

According to the 2007 SCF, entrepreneurs represent 8.3% of the population. This number has slightly fallen in recent years and was equal to 7.2% in the 2016 SCF. As previous studies have documented, although entrepreneurs represent a small fraction of the population, they earn 22% of total income and hold 35% of total wealth. In our calibration strategy we will target these key features about the role of entrepreneurs in the economy.

Throughout our analysis we assume that each entrepreneur owns and manages only one firm. This implies that the number of firms traded every period is equal to the number of entrepreneurs that trade their firm and, hence, we use both terms interchangeably. This assumption relies on the fact that, according to the SCF, more than 80% of entrepreneurs own only one firm. Additionally, according to the SBO, more than 75% of the private firms in the economy have only one entrepreneur, while more than 96% of the firms have at most two.⁵

2.3 How do Entrepreneurs Acquire Their Firms?

Share of Traded Firms. As a first step in our analysis, we focus on the SBO and the 2007 SCF and look at how do entrepreneurs acquire their firms.⁶ Table 1 presents the share of entrepreneurs that: founded their firm, that purchased it, and the share that

⁵In Appendix A.3.1 we document in further detail how the ownership and management of privately held firms in the U.S. is highly concentrated. This is true even for the oldest and the largest private firms in the economy.

⁶Specifically, the SBO asks: "How did [the owner] initially acquire ownership of this business?". Similarly, the SCF asks business owners: "How did you first acquire this business?".

inherited it or acquired it through any other way. The first set of rows present the results for our baseline definition. The second set of rows present the results for firms with positive employment.

Table 1 shows that more than 70% of entrepreneurs founded their firm. This is, of course, the most common way in which entrepreneurs acquire their firms. Also, it shows that between 6 to 10% acquired it through inheritance or other type of acquisition. The most relevant number for our analysis is that 17% of the entrepreneurs acquire their business by purchasing an existing firm. As expected, this number is even larger, between 23 and 25%, for employer firms.

An important thing to note is that the numbers we obtained for entrepreneurs' type of acquisition from the SBO and the 2007 SCF align remarkably well. Particularly the results for the share of entrepreneurs that purchased their business. In the light of this result, we are more confident on using the SCF to measure entrepreneurs' type of acquisition across time.

In Appendix A.2.1 we show that our results are robust to alternative definitions of who is an entrepreneur. We also show that these numbers are not driven by franchises or some specific sectors of production. Overall, across all our robustness exercises, we find that the fraction of entrepreneurs in the U.S. that acquired their firm through a purchase is, roughly, 20%.⁷

Trade of Firms Across Time. As the PUMS version of the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firm across time. As a robustness, we also consider data from the ASE which is available for the years 2014 to 2016. The definition of an entrepreneur in the ASE is closer to the one of an entrepreneur with a positive number of employees.

Figure 1 shows that between 1989 and 2016 the stock of traded firms almost halved. Specifically, considering our baseline definition, the share of traded firms fell by around 10 p.p. going from above 25% in 1989 to a level close to 15% by 2016. In Appendix A.2.3 we show that the decreasing trend is robust to alternative samples and definitions. It is worth mentioning that most of the fall occurred before 2007. Since the Great Recession this share has been relatively stable. In Section 6 we study, through the lens of our model, how looser credit conditions may explain part of the decline in the share of firms traded.

Firms' Trade Rate. The previous results refer to the *stock* of firms that have been traded at any point in the past. We are also interested in the frequency at which firms are traded, i.e., the trade *rate.* We estimate the percentage of firms traded every year

⁷In Appendix A.3.2 we analyze whether the trade of firms is related to entrepreneurs' life cycle. We find that, at most, 10% of the total trades we observe could be related directly to retirement motives.

using two strategies. The first strategy looks at the percentage of firms purchased in the SBO data in the same year of the survey. The second strategy relies on a back of the envelope calculation using the stock of traded firms together with firms' entry and exit rates.⁸ Both strategies, imply that around 2.1% of the firms are traded every year. If we restrict to the sample of firms with positive employment, we obtain an annual trade rate of 3%.

2.4 Firm Buyers' Previous Occupation

From the SBO we can obtain information regarding entrepreneurs' previous occupation. We found that between 62 and 66% of the entrepreneurs that purchased their firm, for our baseline and secondary definitions respectively, have never been self-employed. Hence, most likely, these individuals were in the labor market before acquiring their firm.⁹ This result indicates that purchasing an existing firm is a relevant channel for entering into entrepreneurship which, to the best of our knowledge, has not been studied before.¹⁰ Our theoretical framework will incorporate this possibility of transiting into entrepreneurship through the market for firms.

2.5 Trade Rate and Firms' Characteristics

In this last section of the empirical analysis, we document trade rates conditional on firms' observable characteristics. The study of these relations is important as they are informative about the underlying mechanisms behind the trade of firms. We focus on three main characteristics: firms' age, size, and the average product of capital.

Firm Age. We measure firms' age using data from the SBO. Specifically, we look at all the businesses purchased in 2007 and compute firms' age as the difference between the year the firm was purchased (2007) and the year when the firm was founded. Panel (a) of Figure 2 presents the trade rate across different age bins. The figure shows that startups, defined as firms with less than a year old, are the ones more likely to be traded and have a probability of trade which is 5 times larger than the trade rate of older firms.

Firm Size. We also use the SBO to study the relation between trade and size, but now we focus on the side of the seller as we want to measure firm size before trade takes place. For this, we look at the sample of business owners that sold their firm in or after 2007 and measure size using data from the previous year of operation. Thus, we relate the

⁸See Appendix A.4 for the details of these calculations.

⁹The exact question of the SBO we consider is: "*Prior acquiring this business, had the owner ever* owned a business or been self-employed?" Given the design of the question, these numbers should be interpreted as lower bounds for what would be our non-entrepreneur definitions (i.e. the complement of being an entrepreneur).

 $^{^{10}}$ In Appendix A.2.2 we show that this result is robust to alternative samples and definitions. Further, we show that the share of workers among firm buyers does not appear to be related to specific types of firms.

probability of trade at t against the size of the firm at t - 1. We consider two measures of size given by firm's total sales or total payroll. Panels (c) and (d) of Figure 2 present the probability of trade for different quintiles of the sales and payroll distributions. We find that smaller firms, measured by either sales or payroll, are the ones with the highest probability of trade.

Returns to Capital. Finally, we document the relation between the trade rate and firms' average product of capital (APK). We measure APK using data from the KFS as, different from the SBO, this data includes information about firms balance sheets that allow us to compute a firm-level measure of capital. We measure capital as the sum of the book value of inventories, equipment and machinery, land, buildings, and structures, vehicles, and other type of assets owned by the business. As the analysis for size, we relate firms' APK at period t - 1 against the probability of trade at t, which we measure as the share of owners that report to have sold or merged their business. Panel (b) of Figure 2 shows that high APK firms are the ones with the highest probability of trade.

3 A Model of Entrepreneurship and Trade of Firms

In this section we develop a heterogeneous agent model of entrepreneurship with a *frictional* market for firms that accommodates the previous empirical results.

3.1 Environment

Our model economy is inhabited by a continuum of households in [0, 1]. Households can be either: *firm owners* or *workers*. Firm owners can buy and sell firms, and choose whether to operate their current firm and be *entrepreneurs*, or close the firm and become workers. Workers can become firm owners by acquiring a firm, or through some exogenous *startup* shock. The transitions between these two types of agents are explained in further detail below.

Besides the firms managed by households, which we call *private firms*, there is a second sector of production that features a representative *public firm*. Both sectors produce the same good which can be used for consumption or for savings. Capital is produced by a *financial intermediary* which, each period, takes savings from households and rent capital to the firms. The public firm and the financial intermediary are owned by all households in equal shares.¹¹

Time is discrete and infinite, and each time period is divided in two stages. In the first stage, which we call the decentralized market, or DM, is when the trade of firms

¹¹Alternatively, we could have assumed that the intermediary and the public firm issue equity shares which are traded between households in a frictionless centralized market. This setup is analogous, as assets and shares holdings would be indeterminate. Below we assume that the intermediary and the public firm make zero profits, thus, this modeling choice is not crucial for the analysis.

takes place. We assume that, in the market for firms, households meet bilaterally subject to *search frictions* which may restrain the frequency and the type of the matches. In the second stage, which we call the centralized market, or CM, is when all production, consumption and saving decisions take place.

3.1.1 Households

Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient.

They are heterogeneous in their occupation and in their asset holdings a. Assets are subject to a non-borrowing constraint, $a \ge 0$, and are deposited with the financial intermediary which pays a risk-free interest rate r for the deposits. There is no aggregate uncertainty in this economy, however, households face idiosyncratic uninsurable risk.

Firm owners are endowed with a private firm which enables the owner to produce the final consumption good with a technology that uses capital, labor, and the quality of the firm. This technology is described below. The quality of the firm, denoted by z, is stochastic and evolves according to the law of motion

$$z' = \begin{cases} z & \text{with pr. } \gamma \\ z' \sim \mathcal{P}(z_{min}, \eta_z) & \text{with pr. } (1 - \gamma) \end{cases}$$

where \mathcal{P} denotes a Pareto distribution with scale and a shape parameters z_{min} and η_z , respectively. The $(1 - \gamma)$ shock can be interpreted as changes in market conditions that affect the profitability of entrepreneurial projects as in Buera, Kaboski, and Shin (2011).

On the other hand, workers are endowed with one unit of labor, which they supply inelastically, and are heterogeneous in their labor efficiency ε . We assume that the logarithm of the labor market efficiency evolves according to an AR(1) process with persistence ρ_{ε} and volatility σ_{ε} . Specifically,

$$\log \varepsilon' = \rho_{\varepsilon} \log \varepsilon + \sigma_{\varepsilon} u,$$

where u is a standard normal random variable.

Regarding the transitions between occupations, workers can become firm owners by purchasing an existing firm or through an exogenous *startup* shock at the end of the period. At the beginning of the production stage, firm owners face an occupational choice. They decide whether to operate their firm, and be entrepreneurs, or shut down the firm and become workers. Upon exit or upon selling, previous firm owners lose the value of their firm and enter to the labor market with the lowest labor market efficiency $\underline{\varepsilon}$.¹² We interpret this low value of entry as potential costs associated with entrepreneurship, for example, lack of experience in the labor market.¹³ A graphical description of the transitions between occupations is presented in Figure 3.

In this setup, the budget constraint of an entrepreneur, defined as a firm owner that decides to operate, with states (a, z) is given by

$$c = \pi(a, z) + (1 + r)a - a' + \Pi^{c} + \Pi^{f}$$

and the budget constraint of a worker with states (a, ε) is

$$c = \varepsilon w + (1+r)a - a' + \Pi^c + \Pi^f,$$

where π are the profits of the entrepreneur's private firm, w is the labor market wage, Π^c and Π^f are the public firm and the financial intermediary profits, respectively.

3.1.2 Private Firms

Private firms are endowed with a technology that uses capital k, labor l and the quality of an *entrepreneurial project* z to produce the final consumption good according to

$$y = zk^{\theta}l^{\iota}$$

where $\theta + \nu < 1$. The decreasing returns to scale assumption implies that all private firms have an optimal operation scale as in Lucas (1978).

Private firms rent capital and hire workers every period, hence, they are characterized only by the quality of z. Private firms are indivisible, rival, and excludable. This is an important distinction between our model of trade of firms and the literature that studied trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016).¹⁴ Different valuations of z aim to capture differences in firms' *intangible assets*. For example, trademarks, patents, processes, permits, customer bases, etc.

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm to produce at their optimal scale. Specifically, we assume a collateral constraint that limits the borrowing capacity of the firm to a multiple of the owner's assets which is

¹²Although the distribution of ε is bounded below by 0, in our numerical solution we take $\underline{\varepsilon}$ to be the lowest value on the ε grid, which is a positive number.

¹³There is also a technical reason for why we assume that firm owners that exit into the labor market start with $\underline{\varepsilon}$. If this wasn't the case, and hence suppose they get a value $\tilde{\varepsilon}$, workers with $\varepsilon < \tilde{\varepsilon}$ would have an incentive to buy a low-quality firm and then immediately exit just to improve their labor efficiency.

¹⁴By definition, ideas are non-rival. However, ideas might be excludable under certain institutional arrangements such as patents.

parameterized by λ .¹⁵

Given these assumptions, the profit maximization problem of an entrepreneur with assets a and a firm of quality z is given by

$$\pi(a, z) = \max_{k, l} y - Rk - wl$$

s.t.
$$y = zk^{\theta}l^{\nu}$$

$$k \le \lambda a$$
(1)

where R is the capital rental rate.¹⁶ If the collateral constraints binds $(k = \lambda a)$, the firm operates at a lower scale compared to the unconstrained profit maximization level.

3.1.3 Public Firm

As in Cagetti and De Nardi (2006), we assume that there is a second sector of production populated by a representative public, or corporate, firm. This aim to capture the fact that, in the U.S. economy, around half of the total output is produced by publicly traded firms.

Specifically, we assume that the public firm is owned by all households, in equal shares, and faces no financial frictions. The public firm is endowed with a constant return to scale technology

$$Y_c = K_c^{\eta} L_c^{1-\eta}$$

where K_c is the corporate firm capital, L_c is the corporate firm labor, and Y_c is the total output.¹⁷

3.1.4 Financial Intermediary

The financial intermediary takes deposits from households and rent capital to the firms at a price equal to the savings interest rate plus the capital depreciation rate: $R = r + \delta$. We assume that the representative intermediary operates in a perfectly competitive market and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is given by

$$K_c + \int k(a,z) \, \mathrm{d}N^e_{cm}(a,z) = \int a \, \mathrm{d}N^e_{cm}(a,z) + \int a \, \mathrm{d}N^w_{cm}(a,\varepsilon) \tag{2}$$

where N_{cm}^e and N_{cm}^w are cumulative distribution functions for entrepreneurs and workers, respectively, which are normalized such that $\int dN_{cm}^e + \int dN_{cm}^w = 1$. These measures

 $^{^{15}}$ This type of constraint can be microfounded with an imperfect enforcement of contracts problem. Here, we assume that the quality of the entrepreneurial project cannot be used as a collateral.

 $^{^{16}\}mathrm{In}$ Appendix B.1 we present the entrepreneurs' input demand functions that characterize the static solution of this problem.

 $^{^{17}}$ In Section 7 we study an alternative economy without the public firm. This extension aims to capture the notion that publicly-held firms have an entrepreneurial origin.

correspond to the production stage after firm owners' decided whether to be entrepreneurs or workers.

3.2 Timing

The timing of the model can be summarized as follows:

- 1. The startup shocks, the quality of entrepreneurial projects z, and the labor efficiencies ε are realized.
- 2. Agents enter to the market for firms (DM). Firm owners can buy and sell firms, while workers can only buy.
- 3. Agents enter to the production stage (CM). Given prices and their current z, firm owners decide whether to operate the firm or go to the labor market. Finally, production takes place and agents decide how much to consume and save.

3.3 A Market for Firms

Firms are hard to valuate and price. This precludes the existence of a centralized market with a complete price schedule for different types of firms. Therefore, we model the market for firms using a *search-theoretic* approach characterized by bilateral random matching and *quid pro quo* trade (no credit). An interpretation of this setup is that agents can valuate only one firm at a time, delaying trade.

Trade in the market for firms consists in the transfer of both the ownership and the management of the firm in exchange of assets. Hence, the media of exchange in these transactions are the households' savings a. As we assume that firms are indivisible, when a buyer and a seller meet they only bargain over the selling price p.

There are two types of meetings in the market for firms: *owner-owner* meetings and *owner-worker* meetings. We allow for different search frictions in each type of meeting. Note that firm owners are the only potential sellers while both types of households can be buyers. This implies that in an owner-worker match the owner is the potential seller and the worker is the potential buyer. However, in the case of an owner-owner match who is the buyer and who is the seller depends on the relative quality of the firms.

Let us first consider the owner-owner match and suppose that $z < \tilde{z}$. Then, the owner with states $\mathbf{s}^o \equiv (a, z)$ is the potential buyer and the owner with states $\tilde{\mathbf{s}}^o \equiv (\tilde{a}, \tilde{z})$ is the potential seller. This follows from the key assumption that households can own only one firm at a time, hence no owner would buy another firm which has a lower quality. In this match, the total surplus from trading the ownership of firm \tilde{z} , in exchange of p assets, is given by

Total surplus
$$\equiv \underbrace{W^{o}(a-p,\tilde{z}) - W^{o}(\mathbf{s}^{o})}_{\text{Buyer's surplus, } S_{b}} + \underbrace{W^{w}(\tilde{a}+p,\underline{\varepsilon}) - W^{o}(\tilde{\mathbf{s}}^{o})}_{\text{Seller's surplus, } S_{s}}$$
 (3)

where W^o and W^w are the value functions at the beginning of the production stage for firm owners and for workers, respectively. Upon selling the household goes to the labor market with labor efficiency $\underline{\varepsilon}$, as presented in the first term of the seller's surplus.¹⁸ The outside option for both agents (the terms with a minus in the surpluses) is the value of going to the production stage as firm owners with their initial states \mathbf{s}^o and $\tilde{\mathbf{s}}^o$, respectively.

Regarding the owner-worker match, suppose that a firm owner with states $\tilde{\mathbf{s}}^o$ meet with a worker with states $\mathbf{s}^w \equiv (a, \varepsilon)$. Then, the total surplus from trading firm \tilde{z} is now given by

Total surplus
$$\equiv \underbrace{W^{o}(a-p,\tilde{z}) - W^{w}(\mathbf{s}^{w})}_{\text{Buyer's surplus, } S_{b}} + \underbrace{W^{w}(\tilde{a}+p,\underline{\varepsilon}) - W^{o}(\tilde{\mathbf{s}}^{o})}_{\text{Seller's surplus, } S_{s}}$$
 (4)

where the only difference with respect to the previous match is the buyer's outside option. In this case, if the parties don't trade, the buyer would continue to the production stage as a worker with its initial state \mathbf{s}^w .

Let \underline{p} denote the minimum price at which the seller is willing to sell the firm, i.e. the price at which the seller's surplus is equal to zero. Likewise, let \overline{p} be the maximum price that the buyer is willing to pay for the firm, i.e. the price at which the buyer's surplus is equal to zero. A sufficient condition for trade to occur, meaning that there are positive gains from trade, is that

$$\underline{p}\left(\tilde{\mathbf{s}}^{o}\right) < \overline{p}\left(\mathbf{s}, \tilde{z}\right) \tag{5}$$

where $\mathbf{s} \in {\{\mathbf{s}^o, \mathbf{s}^w\}}$ in function of the type of match (owner-owner or owner-worker, respectively).

In the case that there are positive gains from trade, we assume that the price is determined by a Nash bargaining protocol. Thus, for $\mathbf{s} \in {\mathbf{s}^o, \mathbf{s}^w}$, the trading price p solves

$$p(\tilde{\mathbf{s}}^{o}, \mathbf{s}) = \arg \max_{p} \left[S_{b}(\mathbf{s}, \tilde{z}, p) \right]^{\chi} \left[S_{s}(\tilde{\mathbf{s}}^{o}, p) \right]^{1-\chi}$$

s.t. $S_{b}(\mathbf{s}, \tilde{z}, p) \ge 0, \ S_{s}(\tilde{\mathbf{s}}^{o}, p) \ge 0$ (6)

¹⁸One could think that, if z is very low, some firm owners might even want to pay for someone to buy their firm, implying p < 0, to be able to transition into the labor market. The free exit assumption, through which firm owners can decide to exit and get the same labor efficiency $\underline{\varepsilon}$, rules out the possibility of negative prices.

where S_b and S_s are the buyer and seller surpluses, defined in (3) and (4), and χ the parameter determining the buyer's bargaining power. Throughout the paper we assume that the buyer has all the bargaining power. Thus, we study the case where $\chi = 1$.¹⁹

3.4 Recursive Formulation

We now present the recursive problem of firm owners and workers. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

3.4.1 Value at the Market for Firms (DM)

Firm owners have four potential outcomes upon entering to the market for firms: (1) they don't trade, (2) they buy a firm, (3) they sell their firm to another owner, and (4) they sell their firm to a worker. The no trade case could arise because the owner did not match with a counterpart, or because there was a match but it did not end with a trade.

The value at the beginning of DM for firm owners is given by

$$V^{o}(a, z) = \Pr^{o} [\text{ no trade } | a, z] W^{o}(a, z) \qquad (\text{no trade}) \\ + \alpha_{o} \int_{z < \tilde{z}, \ \underline{p} < \overline{p}} W^{o}(a - p, \tilde{z}) dN^{o}_{dm}(\tilde{a}, \tilde{z}) \qquad (\text{buy}) \\ + \alpha_{o} \int_{z > \tilde{z}, \ \underline{p} < \overline{p}} W^{w}(a + p, \underline{\varepsilon}) dN^{o}_{dm}(\tilde{a}, \tilde{z}) \qquad (\text{sell to a firm owner}) \\ + \alpha_{w} \int_{\underline{p} < \overline{p}} W^{w}(a + p, \underline{\varepsilon}) dN^{w}_{dm}(\tilde{a}, \tilde{\varepsilon}) \qquad (\text{sell to a worker})$$

$$(7)$$

where α_o and α_w are exogenous matching probabilities conditional on each type of match.²⁰ These parameters, both in [0, 1], govern the degree of search frictions in the market for firms. N_{dm}^o and N_{dm}^w are cumulative distributions for firm owners and workers at the beginning of DM, which satisfy that $\int dN_{dm}^o + \int dN_{dm}^w = 1$.

¹⁹In Appendix C.4 we study an alternative distribution of the trading surplus. Our main results remain qualitatively unchanged when we consider the opposite extreme case in which the seller has all the bargaining power ($\chi = 0$). For computational and expositional purposes, we consider $\chi = 1$ for our baseline results.

²⁰In more detail, the probabilities of the bilateral meetings in (7) are derived as follows. First, note that there is a mass $\int dN_{dm}^o$ of owners at the beginning of DM. This implies that two owners are matched with probability $\int dN_{dm}^o$. Due to the search friction, conditional on the match, these owners meet with probability α_o . Thus, the probability of an owner-owner meeting is equal to $\alpha_o \int dN_{dm}^o$. Similarly, the probability that the owner matches with a worker is equal to $\int dN_{dm}^w = 1 - \int dN_{dm}^o$, and conditional on the match they meet with probability α_w . Hence, the probability of an owner-worker meeting is equal to $\alpha_w \int dN_{dm}^w$. Finally, note that the no trade probability \Pr^o [no trade | a, z] sums up the probability of no-meetings plus probability of meetings that do not results in a trade as $p < \overline{p}$ is not satisfied.

As mentioned in Section 3.3, for the case of owner-owner meetings who buys and who sells depend on the relative firm qualities. Hence, an owner with firm quality z, might buy if its matched with another owner with a firm of higher quality $(z < \tilde{z})$, as denoted in the integral in the second line of (7). On the contrary, the owner might sell if its matched with another owner with a firm of lower quality $(z > \tilde{z})$ as denoted in the integral of the third line.²¹ Finally, note that the integrals for the buying and selling cases consider only the meetings that result in a trade, captured by the $p < \bar{p}$ condition presented in (5).

For workers there are only two potential outcomes: (1) they don't trade, or (2) they buy an existing firm. Hence, the value for workers at the beginning of DM is given by

$$V^{w}(a,\varepsilon) = \Pr^{w} \left[\text{ no trade } \mid a,\varepsilon \right] W^{w}(a,\varepsilon) \qquad \text{(no trade)} \\ + \alpha_{w} \int_{\underline{p}<\overline{p}} W^{o}(a-p,\tilde{z}) \, \mathrm{d}N^{o}_{dm}(\tilde{a},\tilde{z}) \qquad \text{(buy)}$$
(8)

3.4.2 Value at the Production Stage (CM)

As previously described, at the beginning of the production stage, firm owners face an occupational choice. They have to decide whether to operate the firm, and be entrepreneurs, or shut down the firm and go to the labor market with labor productivity $\underline{\varepsilon}$. Given these assumptions, the value of firm owners at the beginning of CM is

$$W^{o}(a,z) = \max_{a} \left\{ W^{e}(a,z), W^{w}(a,\underline{\varepsilon}) \right\}$$
(9)

where e denotes the owners' occupational choice.

The value function of entrepreneurs is given by

$$W^{e}(a, z) = \max_{a', c} \quad u(c) + \beta \left\{ \gamma V^{o}(a', z) + (1 - \gamma) \mathbb{E}_{z'} \left[V^{o}(a', z') \right] \right\}$$

s.t. $c = \pi(a, z) + (1 + r)a - a'$
 $c \ge 0, \ a' \ge 0$ (10)

and the value function of workers by

$$W^{w}(a,\varepsilon) = \max_{a',c} \quad u(c) + \beta \left\{ \zeta \mathbb{E}_{\varepsilon'|\varepsilon} \left[V^{w}(a',\varepsilon') \right] + (1-\zeta) \mathbb{E}_{z'} \left[V^{o}(a',z') \right] \right\}$$

s.t. $c = \varepsilon w + (1+r)a - a'$
 $c \ge 0, \ a' \ge 0$ (11)

where $(1-\zeta)$ represents the exogenous startup shock through which a worker can become

²¹Meetings in which the owners have the same firm quality $(z = \tilde{z})$ do not result in a trade.

a firm owner.²²

3.5 Equilibrium

A competitive equilibrium in this economy consists of: (i) aggregate prices $\{r, w\}$; (ii) terms of trade in the market for firms given by the prices of owner-owner meetings $(p, \underline{p}, \overline{p})_{\{\tilde{\mathbf{s}}^o, \mathbf{s}^o\}}$ and owner-worker meetings $(p, \underline{p}, \overline{p})_{\{\tilde{\mathbf{s}}^o, \mathbf{s}^w\}}$; (iii) a decision rule for firm owners' occupational choice e(a, z); (iv) consumption and savings decisions for entrepreneurs $\{c(a, z), a'(a, z)\}$ and for workers $\{c(a, \varepsilon), a'(a, \varepsilon)\}$; (v) capital and labor demand functions for private and public firms, $\{k(a, z), l(a, z), K_c, L_c\}$; and (vi) measures of agents over types and idiosyncratic states at DM and CM subperiods characterized by $\{N_{dm}^o(a, z), N_{dm}^w(a, \varepsilon)\}$ and $\{N_{cm}^e(a, z), N_{cm}^w(a, \varepsilon)\}$, respectively, such that:

- 1. In DM, the terms of trade in bilateral meetings are solved by the bargaining problem.
- 2. In CM, given prices, households, private and public firms solve their optimization problems.
- 3. Goods market clears, period by period:

$$Y = C + K' - (1 - \delta)K$$
(12)

where

$$Y \equiv Y_c + \int zk(a, z)^{\theta} l(a, z)^{\nu} dN_{cm}^e(a, z)$$
$$C \equiv \int c(a, z) dN_{cm}^e(a, z) + \int c(a, \varepsilon) dN_{cm}^w(a, \varepsilon)$$
$$K \equiv K_c + \int k(a, z) dN_{cm}^e(a, z).$$

4. Labor market clears, period by period:

$$L_c + \int l(a,z) \, \mathrm{d}N^e_{cm}(a,z) = \int \varepsilon \, \mathrm{d}N^w_{cm}(a,\varepsilon).$$
(13)

- 5. The budget constraint of the financial intermediary, specified in (2), is satisfied period by period.
- 6. The measures over types and states satisfy

$$\int \mathrm{d}N^o_{dm}(a,z) + \int \mathrm{d}N^w_{dm}(a,\varepsilon) = 1$$

²²In (10) and (11) we omit the corporate firms and financial intermediary profits (Π^c and Π^f terms) in the households' budget constraints as both terms are equal to zero, in equilibrium.

$$\int \mathrm{d}N^e_{cm}(a,z) + \int \mathrm{d}N^w_{cm}(a,\varepsilon) = 1$$

and are consistent with a recursive equilibrium mapping dictated by firms' prices, households' optimal choices, and the stochastic processes for firms' qualities and workers' labor efficiencies.

We solve the stationary equilibrium of this model by approximating the value function using projections methods. The details of our numerical solution are presented in Appendix B.2.

4 Model Properties

In this section we describe the main properties of our model. First, we describe how financial frictions generate motives for the trade of firms. Second, we characterize who buys and who sell firms in our economy. Finally, we describe the implications of this market for firm dynamics and for the allocation of capital.

4.1 Gains from the Trade of Firms

Given that trade in the market for firms is voluntary, a necessary condition for trade is that agents have different valuations for the same firm. In particular, the buyer needs to have a higher valuation than the seller. In our model, two ingredients generate heterogeneous valuations for firms and, hence, potential gains from trade: *firms' credit constraints* and *incomplete markets*. We explain the intuition behind these two frictions through two different channels: the *credit channel* and the *risk channel*.

Credit Channel. The first channel arises from the collateral constraint in the entrepreneurs' problem, presented in (1), which might restrain the firm from producing at the optimal scale. Consequently, whenever an entrepreneur is credit constraint, a wealthier buyer would obtain a higher profit stream out of the same firm as it would be able to operate at a level closer to the optimal scale. Thus, the presence of credit constraints in this economy implies that there are potential gains from the trade of firms between constrained business owners and wealthier buyers. It's worth mentioning that as $\lambda \to \infty$, the profits stream of a firm, $\pi(a, z)$, will not depend on the owner's assets and hence there will be no gains from trade through this channel.

Risk Channel. To analyze this channel let us assume that $\lambda \to \infty$, hence there are no gains coming from heterogeneous profits' streams. Because of incomplete markets, operating a firm comes with an uninsurable risk as its quality z is stochastic. This uninsurable risk generates precautionary savings. An agent can increment its savings either by delaying consumption or, in the case of firm owners, by selling their firm. Further, as wealth increases the precautionary motive is reduced which, in turn, lowers the potential benefits from selling the firm. Because of this risk channel the value of owning a firm z will vary across the wealth distribution generating potential gains from trade. Indeed, in our calibrated economy unconstrained firms also have a positive probability of trade. However, their trade rate is 4 times smaller compared to the probability of trade of credit constrained firms.

4.2 Who Buys and Who Sells Firms?

We start our characterization of the market for firms by analyzing the prices at which firms are traded. If we assume that the buyer has all the bargaining power the selling price is equal to the sellers' minimum price which, conveniently, only depend on the seller's idiosyncratic states. Specifically, if $\chi = 1$ the firm selling price that results from the Nash bargaining protocol, stated in (6), is equal to

$$p(\tilde{\mathbf{s}}^o, \mathbf{s}) = p\left(\tilde{\mathbf{s}}^o\right)$$

where, as before, $\tilde{\mathbf{s}}^{o}$ denotes the states of the seller and $\mathbf{s} \in {\{\mathbf{s}^{o}, \mathbf{s}^{w}\}}$ the states of the buyer.

Panel (a) of Figure 4 plots the sellers' minimum price in the firm owners' state space (a, z). As one would expect, the price is increasing in the firm quality z. But also, due to the collateral constraint on firm owners' wealth, the price of a firm of quality z is increasing in a. Note that this slope is particularly steep for high-quality firms. Thus, even for the highest quality firms, owners with few assets would be willing to sell their firms at a relatively low price as it will take them a long time, and high saving rates, to grow out of their borrowing constraint through self-financing.

Now, keeping in mind how trading prices are determined, we characterize who buys and who sells firms in our economy. Panel (b) of Figure 4 presents the probability that a firm owner sells its firm, again, in the state space (a, z). The figure shows that owners with low wealth and high-quality firms have the highest probabilities of selling. In those cases, there will be high gains from trade as the current owner does not have enough resources to operate at the optimal scale.

Panels (c) and (d) of Figure 4 present the probability of buying a firm for firm owners, in the (a, z) space, and for workers in the (a, ε) space. These panels show that the probability of buying is the opposite mirror image of the probability of selling. Thus, the buyers of firms are going to be wealthy households that currently own low quality firms (low z), or wealthy workers with low labor efficiency (low ε).

4.3 Implications for Firm Dynamics and Capital Allocation

To provide further intuition about the implications of the trade of firms, Figure 5 presents a hypothetical trajectory of a firm in our model. We assume that the initial owner of the firm is the median worker in the economy who at period 0 receives a good

quality firm through the exogenous startup shock. Because of financial constraints that limit the use of external funding, this entrepreneur will start operating the firm at a very low scale. Panel (a) shows that this business owner will accumulate assets over time to reach the optimal operating scale through self-financing. This entrepreneur, however, will take more than ten years to be able to produce at the optimal, unconstrained, scale as shown in panel (b).

Panels (a) and (b) exemplify the basic mechanism through which financial frictions can generate misallocation of capital and, therefore, low aggregate output and low TFP. As shown by Midrigan and Xu (2014), capital misallocation will be high in economies in which this type of situations, where high quality firms (or high ability entrepreneurs) are credit constrained, are relatively frequent.

Now we analyze what would happen if the initial owner had the possibility to sell its firm. Panels (c) and (d) of Figure 5 plots the selling price and the probability of selling the firm, respectively, for this business owner. Just after receiving the startup shock this entrepreneur will be willing to sell its firm at a relatively low price as the alternative option of self-financing implies a low profit stream for several periods. In addition, because of the risk channel described above, a credit constrained entrepreneur will be willing to sell its firm because of precautionary motives. As the initial owner accumulates assets the seller's minimum price will increase and the probability of trade will fall accordingly.

Suppose now that in period 3 a wealthier household purchases this firm. If the second owner has more resources to invest in the firm, this owner will be able to take the firm closer to its optimal operating scale faster. In our example, panel (b) shows that with the second owner the firm reaches its optimal scale in close to 5 years after being founded. This is half of the time required by the initial owner. In sum, this simple example illustrates how the market for firms allows for a better allocation of productive projects and available resources in the economy. In the following sections we describe the parameterization of our model and then quantitatively evaluate the importance of this mechanism in terms of output and TFP.

5 Parameterization

This section describes our calibration strategy and presents our validation exercise. The model is calibrated at an annual frequency to the year 2007, the year for which we have both the SBO and SCF data available.

5.1 Assigned Parameters

We set the relative risk aversion parameter to $\sigma = 1.5$. The capital depreciation rate is set to $\delta = 0.06$, while the elasticity of capital in the public firm production function is set to $\eta = 1/3$. As mentioned above, we assume that the buyers have all the bargaining power, hence, we set $\chi = 1$. These parameters are summarized in Table 2.

5.2 Calibrated Parameters and Targeted Moments

The remaining parameters are chosen such that the model replicates several features of the U.S. economy. To reduce the dimension of the parameter space we assume that private firms' technology has the same relative elasticity between capital and labor as the public firm. In such a way, the decreasing returns to scale in firms' technology are captured by a single parameter $\Upsilon < 1$ such that $\theta = \eta \Upsilon$ and $\nu = (1 - \eta) \Upsilon$.

After this we have a total of 11 parameters which we calibrate to match 15 moments. These parameters, together with their calibrated values, are presented in Table 3. We find those values by minimizing the weighted distance between the moments in the data and in the model. The 15 moments we consider are listed in Table 4. For easier exposition we divide these moments in five groups which are now described.

The first group of moments are related to the role of entrepreneurs in the economy. As reported in the 2007 SCF, we target that 8.3% of households are entrepreneurs, and that they earn 22% of total income and hold 35% of the total wealth. Our second set of moments are about the income and wealth distribution, both across all households and within workers and entrepreneurs. We target six different Gini indexes which we also compute from the 2007 SCF. The table shows that our model does a good job in matching the dispersion of wealth observed in the data, however, it falls short in terms of matching the level of income inequality. It is worth mentioning that different from the previous literature which have abstracted from firm prices, our definition of wealth includes the value of private firms (a + p in the model).

The third and fourth set of moments capture relevant characteristics of firms. First, we target a capital to output ratio of 3. Second, we target that private firms account for 44% of total output, which is the middle point in the estimates presented by Dinlersoz et al. (2019). This number is relatively conservative compared to Asker, Farre-Mensa, and Ljungqvist (2014) who estimate that private firms account for 57% of total sales. Regarding private firms leverage we target the weighted average debt to assets ratio in our model to be 0.35 which we obtain from the noncorporate firm leverage in the Flow of Funds Accounts. This number is consistent with the firm-level evidence presented in Crouzet and Mehrotra (2020). We also target a firm exit rate of 0.08, which is in line with the numbers in literature on firm dynamics, for example as in Hopenhayn, Neira, and Singhania (2018).

Our fifth and final set of moments capture two relevant features about the trade of private firms which we documented in the empirical section of the paper. Specifically, that 2.1% of the firms are traded every year and, from those, 62% of them are acquired by workers who enter to entrepreneurship by purchasing an existing firm. Table 4 shows

that our model does a good job matching the targeted moments. Particularly the ones related to entrepreneurs, private firms, and the market for firms, to which we assign a higher weight in our calibration exercise.

5.3 Validation: Financial Frictions as a Motive for Trade

As has been described throughout the paper, if financial frictions are an important motive for the trade of firms, credit constrained firms should be the ones more likely to be bought and sold. We test this prediction of the model by analyzing the relation between trade and firms' observable characteristics. As in the empirical section, we consider two commonly used proxies of credit constraints: firms' age and firms' size as younger and smaller firms are more likely to be financially constrained. In addition, we analyze firms' APK since credit constrained firms will have high returns to capital, but they are not able to increase their investment.

Following the analysis presented in Section 2.5, we simulate data from our model and compute the firms' trade rate conditional on these characteristics. Figure 2 shows that, consistent with the data, our model predicts that younger, smaller, and high returns to capital firms present the highest probabilities of trade. Even though our model implies relatively starker relations, of these variables and the trade rate, it qualitatively aligns well with the data.

It is important to remember that these relations were *not* targeted in our calibration exercise. Rather, they result from the key prediction that credit constrained firms are the ones more likely to be traded, and by the fact that these characteristics are strongly correlated with binding credit constraints in our model. Overall, these results suggest that financial frictions are indeed an important motive for the trade of private firms.

5.4 Other Untargeted Moments

As shown by Quadrini (2000) and Cagetti and De Nardi (2006), among others, an important feature of heterogeneous agents models with entrepreneurship is that they can replicate the income and wealth distribution observed in the data. This is possible thanks to the combination of uninsurable income risk and stochastic returns to wealth coming from entrepreneurial activity. Table 5 shows that this is also true in our model. Although we only targeted a set of Gini coefficients, the model does a good job matching the complete income and wealth distribution observed in the data.

6 Quantitative Analysis

This section presents our main quantitative exercises. First, we present two counterfactual experiments aimed to quantify the relevance of the market for firms as a mechanism through which entrepreneurial projects and available resources are allocated. Second, we analyze the relation between the economy credit conditions and the trade of firms.

6.1 The Role of the Market for Firms

We consider two counterfactual experiments. In the first experiment we take our baseline model and analyze what would occur if the market for firms partially, and then totally, shut downs. In the second experiment we consider an alternative economy with no trade in the ownership of firms. We then analyze how much the access to external financing must improve in order for this economy to match the TFP level of the entrepreneurial sector in our baseline economy with trade. In both experiments we make steady state comparisons of our model under different parameterizations. The results presented in this section show that trade in the ownership of firms is a relevant mechanism through which entrepreneurial projects and resources are allocated.

6.1.1 Closing the Market

Table 6 presents the results of our first counterfactual experiment. As a reference, the first column of the table presents some relevant moments of our baseline economy. The second and third columns report the percentage change in these moments when the market for firms partially and then, completely shut downs. In both cases, we only vary the parameters governing the trade of firms while maintaining the rest of the parameters fixed.

The parameters that determine the search frictions in the market for firms are α_o and α_w , which capture the probability of meeting conditional on the type of match (ownerowner or owner-worker). For the partial shutdown case we divide in half both parameters such that their relative values are the same and, hence, the fraction of firms purchased by workers is unchanged. For the complete shutdown case we set both parameters equal to zero.

In both cases, entrepreneurial output considerably falls: -2.7% and -5.6% for the partial and the complete shutdown case, respectively. For easier exposition we focus on the total shutdown results. The remaining rows of Table 6 shows that the fall in entrepreneurial output is explained by both extensive and intensive margins. First, regarding the extensive margin, there are fewer entrepreneurs producing (-5.2%). Second, the remaining private firms will exhibit a poorer allocation of capital and firms' qualities as shown by the larger number of constrained firms (12.6%), the fall in entry and exit (-21.5%), and the resulting lower entrepreneurial TFP (-1.5%).

Something that stands out from these results is that the fall in total output is quantitatively small (-0.5%). This is explained by how we model the public firm, which is basically a residual of the entrepreneurial sector. Therefore, the fall in the entrepreneurial output is matched by an increase in the output of the representative public firm (3.4%, keeping in mind the relative sizes of the two sectors). In Section 7 we consider an alternative economy with only entrepreneurial firms. This economy will feature a larger fall in total output, compared to our baseline result.

Finally, is worth mentioning what are the implications for the distribution of income and wealth in the economy. Consistent with the fact that entrepreneurs earn, and hold, relatively higher income and wealth shares, shutting down the market for firms disproportionally affect these of agents the most. This will result in a lower income and lower wealth inequality, as measured by the Gini indexes for income and wealth (-0.3% and -3.8%, respectively).

6.1.2 Baseline vs. No Market Economy

For our second experiment we consider an alternative economy with $\alpha_o = \alpha_w = 0$ which we recalibrate such that it matches the same moments as our baseline economy (except for the moments regarding the trade of firms). We call this alternative model the "No Market Economy". In Figure 6 we present different steady states for the baseline and for the no market economy under alternative credit market frictions which, in the model, are governed by the parameter λ . Higher λ implies easier access to credit as entrepreneurs can borrow more with the same level of assets. From these steady states, we focus on three moments: the fraction of entrepreneurs (Panel a), entrepreneurs' debt to capital ratio (Panel b), and the entrepreneurial sector TFP (Panel c).

As the two upper panels of Figure 6 show, the baseline and the no market economy exhibit practically the same relation for both the fraction of entrepreneurs and their debt to capital with respect to λ . Thus, point A in panels (a) and (b) shows that for the same level of λ both economies deliver, roughly, the same value for these two moments. However, this is not the case for the entrepreneurial sector TFP. Panel (c) shows that for the same level of λ the no market economy implies a lower TFP. This is explained by the higher *misallocation* between entrepreneurial projects and available resources.

With these relations at hand, we *ask*: what are the credit conditions that the no market economy requires such that it matches the TFP level of our baseline economy? Using panel (c) we can recover the level of λ such that the no market economy delivers the same level of TFP. Graphically, this implies moving from point A to point B. As B is to the right of A this means that the no market economy requires a higher λ , thus *easier credit* conditions. To better interpret this, we go back to panels (a) and (b) and recover the fraction of entrepreneurs and debt to capital implied by point B. These panels show that the no market economy requires that the fraction of entrepreneurs go up by around 2 p.p., while the debt to capital ratio increase by 7 p.p., or 19%. This is a sizable number as, for example, during the Great Recession firm leverage fell by around 5 p.p.

Overall, these counterfactual exercises show that the market for firms is a relevant

mechanism through which entrepreneurial projects, or investment opportunities, and available resources can be better allocated in the economy.

6.2 Credit Conditions and the Trade of Firms

In the empirical section of the paper, we documented that the share of entrepreneurs that purchased their firm almost halved, going from 25% in 1989 to a level close to 15% by 2016. One implication of our theory is that as credit conditions are looser, and current firm owners have more access to external finance, the gains from trading firms will be smaller, which will reduce the total number of trades in the economy. In this section we test how much of the fall in share of traded firms can be attributed to firms' access to external financing.

Importantly, during the same period in which the fraction of firms purchased fell there was also a secular increase in private firms' leverage, suggesting easier access to credit. Specifically, panel (a) of Figure 7 shows that during this time firm leverage increased by around 20 p.p.²³ As a reference, Panel (b) plots the time series for the stock of firms traded in the SCF, which we described in the empirical section.

Now we analyze the relation between these two time series through the lens of our model. Specifically, we compare two steady states that deliver the same levels of debt to capital, as observed in the data, and compare their implied share of traded firms. We graphically perform these steps in the lower panels of Figure 7. First, from Panel (c) we obtain points A and B such that the model delivers the same levels of debt to capital as observed in data. Then, for those values of λ we go to Panel (d) and see what the change in the volume of traded firms is between points A and B. We find that the fraction of firms purchased falls by around 4 p.p. This result suggests that 40% of the 10 p.p. decrease in the fraction of firms purchased can be explained by the easier access to credit observed during this time period. Although higher access to external financing can account for a substantial share of the drop in the trade of private firms, this result does not rule out that this fall could be related to other trends, such as the decline in business dynamism in the U.S. economy.

7 Robustness and Extensions

In this section we redo our main quantitative exercise, of closing the market for firms, considering alternative parameterizations and extensions from our baseline model. First, we use an alternative calibration strategy that restrict to firms with a positive number of

 $^{^{23}}$ To obtain this number, we consider a linear trend to control for the boom and bust episodes observed in the last decades. Note that the figure starts in the year 1980 as the measure we see in the data for the trade of firms is a stock, not a flow. Hence, to allow for a certain lag in this variable, we consider the credit conditions starting 9 years before our first observation for the stock of firms purchased in the SCF, which is 1989.

employees. Second, we allow for additional motives to trade firms that are orthogonal to financial frictions. Lastly, we consider an economy in which all firms are assumed to have concentrated ownership and are subject to financial frictions.

7.1 Employer Firms

First, we consider a second parameterization for our model that uses our alternative definition of entrepreneurs. In Section 2 we documented that the share of traded firms is larger for this alternative definition of entrepreneurs that requires the firm to have a positive number of employees. With this definition, the implied annual trade rate is 3%, higher than the 2.1% used in our baseline exercise. The higher trade rate suggests that our baseline definition of entrepreneurs might include some independent self-employed individuals with non-tradable capital. In Appendix C.1 we present the details of this alternative parameterization.

The results from completely shutting down the market in the employer firms' model are presented in the second column of Table 7. As expected from the higher trade rate, this model predicts a larger fall in entrepreneurial output equal to -8%. This larger drop is explained by both a stronger reduction in the number of entrepreneurs and higher capital misallocation, as shown by the larger fall in TFP and the sharper increase in the fraction of financially constrained firms.

7.2 Alternative Motives to Trade

In our baseline model, financial frictions are the only motive behind the trade of firms. In the following extension we include alternative motives to trade which are orthogonal to the credit and risk channels studied above. This exercise aims to incorporate alternatives motives to trade firms, such as owners' life cycle (retirement) or migration decisions, in a reduced-form way.

To introduce these alternative motives in a parsimonious manner we assume that upon meeting in the DM subperiod, firm sellers might be of two types. With probability $(1-\psi)$ they are *rational sellers*, which value their firm as before. With probability ψ they are *behavioral sellers* which are willing to sell their firm at a discounted price κp , where $\kappa \in (0, 1)$. The behavioral sellers are compensated with a utility transfer which leaves them indifferent between selling the firm or not. We calibrate these new parameters such that the model matches the trade rate across the quintiles of the firm size distribution (these moments were not targeted in our baseline parameterization). We relegate the details of this alternative model to Appendix C.2.

The third column of Table 7 presents the results of closing the market for firms in the economy with alternative motives to trade. The fall of entrepreneurial output is slightly reduced, but it remains at -5.1%. The drop is also explained by both the extensive and

intensive margins, but now the former appears to be more important than in the baseline case as the fractions of entrepreneurs falls by -5.9% (vs. -5.2% in the baseline). Overall, this shows that our main result, regarding the importance of the market for firms in alleviating financial frictions, is robust to the inclusion of these alternative motives for the trade of firms. This is explained by the fact that the inclusion of these orthogonal motives does not stop the most beneficial trades, between constrained owners and wealthy households, from happening in our model.

7.3 No Public Firms

In our baseline model we assumed that contrary to private firms, the representative public firm is unconstrained and its ownership is sparse. It can be argued, however, that this is not the case for various listed firms. On the one hand, publicly traded firms also pay a premium for external financing, suggesting that they might also be financially constrained. Second, it is not rare that large public firms have concentrated ownership and management.

Motivated by this, in this extension, we drop the representative public firm and assume that all the production in the economy is done by entrepreneurial firms which are subject to financial frictions. For this case, we adjust our calibration strategy accordingly and target a broader definition of entrepreneurs that only requires households to be business owners. Further, we adjust the targeted firms' leverage as the corporate sector features higher leverage, compared to non-corporate firms. The details about this calibration strategy are presented in Appendix C.3.

The results of shutting down the market in the economy without public firms are presented in the last two columns of Table 7. Different from the previous models, we distinguish between the partial and general equilibrium effects (PE and GE, respectively). In PE, keeping prices fixed, total output falls by -3.7%. This fall is mostly explained by the higher misallocation of capital as shown by the -0.7% fall in TFP and the 4.6% rise in the fraction of constrained firms.

This economy, however, features a large GE response following the market shutdown. In particular, the equilibrium interest rate falls by -15.6%. In contrast to the previous models, this leads to an increase in the share of entrepreneurs of 4.8%. Thus, in this model, the extensive and intensive margins go in opposite directions. These opposite forces will lead to a smaller fall in total output, compared to the PE effect, equal to -1.3%. Finally, is worth mentioning that this GE response will lead to an increase on income inequality. This result is explained by the smaller interest rate which, effectively, transfers resources from the savers (workers) to the borrowers (entrepreneurs) of the economy.

8 Conclusions

In an economy with credit market imperfections, constrained entrepreneurs might want to sell their firm to other less constrained parties which will be able to produce closer to the firm's optimal scale. We study this motive behind the trade of firms both empirically and theoretically, and we showed that the trade of firms is a quantitatively relevant mechanism through which entrepreneurial projects and available resources can be better allocated in the economy.

We consider that our paper is a first step towards a better understanding of the role of this market for the aggregate economy. An interesting avenue for future research is to study the role of this market with a richer firms' life cycle structure. This extra structure could create channels through which the trade of firms, for example, may facilitate entry and reduce exit for high-quality firms. These mechanisms are muted in our framework.

Another avenue for future research could be to analyze the relation between the market for firms and technological diffusion. Firms in our paper are assumed to be fully rival, but it may be the case that part of these know-how, or intangible assets, remains with the seller after the transaction. In this sense, partial rivalry of traded firms may work as a channel to create new productive firms and spark growth.

Finally, our paper focuses on the positive aspects, not the normative ones, of the trade of firms. If societal benefits of trading firms are greater than the private ones this may create scope for policies aimed to reduce the trade costs. In this spirit, the careful study of the underlying frictions precluding the trade of firms is crucial. We leave the study of these normative aspects, and these other issues, for future research.

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Figures



Figure 1: Fraction of Entrepreneurs that Purchased Their Business

SOURCE: SBO, SCF and ASE.

NOTES: Entrepreneurs are defined as self-employed, business owners, who actively manage their firm. + Employment > 0 also requires that the firm has a positive number of employees. The light-colored dots correspond to the time series SCF data points. The solid line trend was estimated using locally weighted smoothing.



Figure 2: Trade Rate by Firms' Characteristics

NOTES: Panels (a), (c) and (d) use data from the 2007 SBO. In panel (a) trade is computed using the fraction of owners that acquired their firm through a purchase in 2007. The age of the firm is computed as the difference between 2007 and the year when the business was established. Panel (b) uses data from KFS. Trade is computed using information from all the firms sold during the years of the sample. Average productivity of capital (APK) is measured by sales over capital of the previous year to the sale. Capital includes inventories, equipment and machinery, land, buildings, and structures, vehicles and other assets owned by the business. The relation is computed for every year and then averaged across time. In panels (c) and (d) trade is computed using information from the firms that were sold in or after 2007.





Figure 4: Buyers and Sellers in the Market for Firms



(b) Probability of selling






Figure 5: Firm Dynamics and Trade, An Example

NOTES: The vertical line indicates the DM subperiod at t = 3, when trade takes place. The initial owner has assets equal to the median worker of the economy upon receiving the startup shock at t = 0. In this example firm quality z is held constant across the 15 periods. $\tilde{k}(z)$ in panel (b) denotes the unconstrained optimal level of capital for a firm with quality z. p(a, z) in panel (c) denotes the seller's minimum price.



Figure 6: Baseline vs No Market Economy



Figure 7: Credit Conditions and the Trade of Firms

SOURCE: Flow of Funds Accounts and SCF.

Tables

		Founded	Purchased	Inherited/Other
Entrepreneurs	SBO	77.0%	17.0%	6.0%
	SCF	71.9%	17.7%	10.4%
+ Employment > 0	SBO	65.2%	25.5%	9.3%
	SCF	65.3%	22.7%	12.0%

 Table 1: Share of Entrepreneurs by Business Acquisition

SOURCE: SBO and SCF for the year 2007.

NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

Parameter	Value	Description
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity
χ	1	Buyers' bargaining power

 Table 2: Assigned Parameters

 Table 3: Calibrated Parameters

	Value	Description
β	0.903	Discount factor
Υ	0.707	Curvature private firms technology
λ	2.004	Collateral constraint
γ	0.939	Persistence private firm value
ζ	0.933	1– Startup shock
z_{min}	1.060	Scale, z distribution
η_z	2.523	Shape, z distribution
$ ho_{arepsilon}$	0.944	AR(1) parameter, ε distribution
$\sigma_{arepsilon}$	0.230	Std. Deviation, ε distribution
α_o	0.805	Owner-owner meeting probability
α_w	0.663	Owner-worker meeting probability

	Source	Data	Model
Entrepreneurs			
Fraction of entrepreneurs	SCF	0.083	0.082
Income share of entrepreneurs	SCF	0.22	0.19
Wealth share of entrepreneurs	SCF	0.35	0.40
Income and wealth distribution			
Gini income, all households	SCF	0.62	0.54
Gini wealth, all households	SCF	0.82	0.83
Gini income, entrepreneurs	SCF	0.67	0.70
Gini wealth, entrepreneurs	SCF	0.76	0.81
Gini income, workers	SCF	0.58	0.49
Gini wealth, workers	SCF	0.79	0.79
Private and public firms			
Capital to output ratio	See text	3.0	2.9
Private firms			
Output share	See text	0.44	0.43
Debt to capital ratio	FoF	0.35	0.37
Entry/exit rate	See text	0.08	0.07
Trade of private firms			
Annual trade rate	SBO	0.021	0.021
Share purchased by workers	SBO	0.62	0.61

 Table 4: Targeted Moments

NOTES: Data moments correspond to the year 2007. Wealth in the model is defined as the sum of the risk-free asset and the value of the firm a + p.

	Data	Model		Data	Model
Income distribution, all households			Wealth distribution, all households		
Top 1	0.22	0.16	Top 1	0.33	0.38
Top 5	0.39	0.34	Top 5	0.60	0.61
Top 10	0.49	0.48	Top 10	0.72	0.73
Bottom 75	0.31	0.37	Bottom 75	0.13	0.11
Bottom 50	0.12	0.19	Bottom 50	0.02	0.01
Bottom 25	0.02	0.04	Bottom 25	0.00	0.00
Income distribution, entrepreneurs			Wealth distribution, entrepreneurs		
Top 1	0.22	0.23	Top 1	0.24	0.26
Top 5	0.44	0.63	Top 5	0.48	0.62
Top 10	0.57	0.75	Top 10	0.63	0.80
Bottom 75	0.24	0.16	Bottom 75	0.16	0.10
Bottom 50	0.10	0.10	Bottom 50	0.05	0.04
Bottom 25	0.03	0.10	Bottom 25	0.01	0.04

 Table 5: Untargeted Moments

SOURCE: 2007 SCF.

	$\Delta~\%$		
	Baseline economy	$\frac{\text{Partial}}{(\alpha_o, \alpha_w)/2}$	$\begin{array}{c} \text{Total} \\ (\alpha_o, \alpha_w) = 0 \end{array}$
Fraction of entrepreneurs	0.082	-2.6%	-5.2%
Output private firms	0.52	-2.7%	-5.6%
TFP private firms	1.10	-0.7%	-1.5%
Fraction of constrained firms	0.67	5.6%	12.6%
Entry/exit rate	0.07	-9.8%	-21.5%
Output public firms	0.70	1.6%	3.4%
Total output	1.22	-0.2%	-0.5%
Gini income	0.54	-0.1%	-0.3%
Gini wealth	0.83	-0.3%	-3.8%
Interest rate	0.037	0.8%	1.5%
Wage	1.24	-0.1%	-0.3%

 Table 6: Closing the Market for Firms

NOTES: The Partial column presents the results for the market partial shutdown, obtained dividing by the half the parameters α_o and α_w . The Complete column presents the results when both parameters are equal to zero, thus a total market shutdown. TFP is measured as $Y_e/(K_e^{\theta}L_e^{\nu})$, where $(.)_e$ denotes the aggregate variables of the entrepreneurial sector.

				No Pub	lic Firms
	Baseline	Emp.>0	Alt. Motives	PE	GE
Fr. of entrepreneurs	-5.2%	-7.4%	-5.9%	0.9%	4.8%
Output private firms TFP private firms Fr. of constrained firms Entry/exit rate	-5.6% -1.5% 12.6% -21.5%	-8.0% -1.9% 17.6% -20.0%	-5.1% -1.4% 11.3% -25.8%	-3.7% -0.7% 4.6% -11.5%	-1.3% -0.6% 5.1% -14.8%
Output public firms Total output	3.4%- $0.5%$	4.3% -0.8%	2.7%- $0.4%$	-3.7%	-1.3%
Gini income Gini wealth	-0.3% -3.8%	-0.4% -3.1%	-0.2% -3.8%	-0.6% -2.4%	0.3% -2.5%
Interest rate Wage	1.5% - 0.3%	2.7% - 0.5%	1.7% - 0.3%		-15.6% -0.8%

Table 7: Closing the Market for Firms - Alternative Models and Parameterizations

NOTES: Baseline corresponds to the results for the complete market shutdown in our baseline economy. Emp.>0 denotes the economy calibrated to match the trade of employer firms. Alt. Motives denotes the economy with the orthogonal motives to trade. No Public Firms denotes the economy without public firms. PE denotes the partial equilibrium moments maintaining prices fixed.

A Data Appendix

A.1 Data Sources

A.1.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey of firms and firm owners in the U.S. The PUMS sample is representative of non-farm private businesses with receipts of \$1,000 or more and is available for the year 2007. The SBO is conducted at the company or firm-level. A company is a business consisting of one or more domestic establishments. The survey is designed to identify the ultimate owners of firms and their characteristics.

Table A.1 reports the total number of owners and firms in the SBO. From those, we first restrict to the owners who report how do they acquire their business. The SBO already restricts to self-employed business owners, thus for our definition of entrepreneurs, we just have to restrict to business owners who actively manage their firm. Our baseline sample consist of more than 1.1 million entrepreneurs which own around 840,000 different firms.

	#Dropped	#Owners	#Firms
All	-	3,409,393	$2,\!165,\!680$
Report Acquisition	$6,\!498,\!179$	2,164,541	$1,\!291,\!292$
Entrepreneurs	$1,\!052,\!287$	$1,\!112,\!254$	841,254
+ Employment > 0	413,603	698,651	$501,\!564$

Table A.1: 2007 SBO Sample

From this survey we mainly focus on how the owners acquired their firms. In addition, we use information on the characteristics of the firm (established year, employment, payroll, receipts, sector, location, operation status, number of owners) and of the owners (age, acquisition year, ownership percentage, education level, previous occupation). We use this information to do a thorough characterization of the trade of firms.

Using the SBO we can also obtain information on firms and owners close to the time at which the firm was traded. To study firms' and buyers' characteristics *when purchased* we look at owners that acquired the firm through a purchase in the same year of the survey. Further, the SBO provides information on firms' and owners' characteristics for those owners who report an exit because they sold their firm in the year of the survey. We use this information to characterize firms and their previous owners *when sold*. For all our calculations we use the sample weights provided by the survey.

A.1.2 Survey of Consumer Finances (SCF)

The SCF is a household-level survey that includes extensive information on households' income, balance sheets, and demographic characteristics. The public microdata is available every three years for the period 1989-2016.

	#Dropped	#Households
Income and Wealth		
All	-	47,776
21 < Age < 78	3,528	44,248
Positive Income	67	44,181
Firm Acquisition		
Entrepreneurs	35,474	8,707
+ Employment > 0	1,379	7,328

Table A.2: 1989-2016 SCF Sample

In the SCF we identify entrepreneurs as those households whose household head: is self-employed, owns a business, and has an active management role in it. The SCF also provides information of privately held businesses which are actively managed. Business owners can report information for up to three or two firms, depending on the survey year. For our baseline calculations we focus on the characteristics of the main business, defined as the one with higher reported value. Using this information, we can identify the entrepreneurs that own a firm with a positive number of employees.

Table A.2 reports our sample selection criteria and the number of households in our SCF sample. For our calculations of the moments of income and wealth we restrict to a sample of households whose household head is between 22 and 78 years old and have a positive income. For our calculations of the trade of firms trade we focus on entrepreneurs, which considering our baseline definition, are 8,707 households between 1989 and 2016, which is a significantly smaller than the one in our SBO sample.

Additional to the information on entrepreneurs and how do they acquired their firm, we use the SCF to compute relevant moments from the income and wealth distribution in the U.S. economy. Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real estate, the value of farm and private businesses owned by the household. Debt includes both housing debt (mortgages), debt from lines of credit and credit cards, and installment loans.

Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non-classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities.

For all our calculations we use the sample weights provided by the survey.

A.1.3 Annual Survey of Entrepreneurs (ASE)

The ASE is a representative sample of all non-farm businesses filing Internal Revenue Service (IRS) tax forms as individual proprietorships, partnerships, or any type of corporation, and with receipts of \$1,000 or more. The ASE is conducted at the firm-level and gathers information on the firm and owner characteristics. The population represented by the survey focuses on firms with paid employees. This survey is available at an annual frequency starting in 2014.

Similar to the SBO, the ASE collects information regarding owner' and firms' characteristics for a large sample of owners. The difference is that the ASE has an annual frequency and samples only firms with paid employees. One major caveat of the ASE is that we don't have access to the micro data, therefore we use information from the tables provided by the Census Bureau to compare to our baseline estimates and explore the recent evolution in the share of firms traded.

For the table estimates provided by the Census Bureau, a business owner is defined as someone who holds more than 50% of the stake of the firm, where the firm has a positive payroll. Though this definition is slightly different it is very close to our second definition of an entrepreneur with a positive number of employees. Our numbers are retrieved from table SE1600CSCB001 where entrepreneurs are classified by the way they acquired their firm.

A.1.4 Kaufman Firm Survey (KFS)

The KFS is a panel survey that tracks almost 5,000 business that start their operations in 2004 through 2011. The initial sample was created by using a list frame sample of start-up businesses from the Dun & Bradstreet Corporation (D&B) database. The KFS collects information from business' and owner's characteristics and, in particular, they provide information about firms' balance sheets.

Table A.3 shows the sample selection. Following the previous literature, we drop firms that at some point refuse to answer and observations with missing values of employment, revenues, sales, assets, cash, and accounts receivable. Our baseline sample remains with 2,841 firms and 13,457 observations (firm \times year).

	#Dropped	#Observations	#Firms
All	-	39,424	4,928
Answer	13,624	25,800	3,225
No Missing	12,343	$13,\!457$	2,841

Table A.3: 2004-2011 KFS Sample

We define capital as total assets without cash holdings and accounts receivable. Total assets is composed by product inventories, land and buildings and structures, vehicles, equipment/machinery, other properties, cash, and other. To approximate the capital returns we consider the average productivity of capital (APK) measured as firms' revenue to capital ratio. In the KFS we identify trades as exits due to acquisitions.

For all our calculations we use the sample weights provided by the survey.

A.2 Robustness Exercises

This section presents several robustness exercises for our three main empirical results regarding the trade of firms and firm buyers' previous occupation.

A.2.1 How do Entrepreneurs Acquire Their Firms? - Robustness

Using SBO data we compute the share of business owners that acquired their business through a purchased considering several alternative definitions and restricting to different samples. Our result is robust to these alternative computations. We also compute this share at the firm-level and obtain very similar results. Finally, we show that the share of entrepreneurs that purchased their firm is not driven by franchises or by some specific sector of production.

Owner-level. Table A.4 presents the results for the share of entrepreneurs that acquired their firm through a purchase considering different characteristic of the firm in terms of

its number of employees, size, and payroll. Additionally, we consider alternative definitions of who is an entrepreneur. For example, instead of active management we restrict to business owners who have more than 50% of the equity of the firm, or to owners who work at least 40 hours a week in the firm. In bold we highlight our two baseline definitions for entrepreneurs and entrepreneurs with a positive number of employees. In line with the result of a higher trade share for entrepreneurs with a positive number of employees, Moreover, if we weight entrepreneurs by the employment of their firms the share significantly rises to 32.2%.

Firm-level. Additional to the business owner-level results we look at the share of firms which owners acquired them through a purchase. We compute the share of firms purchased in two ways: (i) if at least one entrepreneur purchased the firm; (ii) equally weighting across entrepreneurs of the same firm. The results are presented in Table A.5. The purchased share computed at the firm or at the owner-level are very similar. This is due to the fact that most firms have one entrepreneur, and most entrepreneurs have one firm. As in the business owner results, this share is sensitive to the exclusion of firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios as the main input in production is probably the owner human capital, which is hard to transfer.

Franchises. We further analyze whether franchises are driving our results. Table A.6 shows that even excluding all franchises the share of entrepreneurs that purchased their firm is 16.1% and 24.1% for our baseline and positive employment definitions, respectively. Although is true that, within franchise owners, the share of entrepreneurs that acquired the business is very high, more than 50%, these owners represent a small group in the total number of entrepreneurs: 2.8% and 4.8% for our two definitions.

Sectors. We also analyze if our results explained by specific sectors of production. The results are presented in Figure A.1. Although there is variability in the stock and rate of trade, we find that the trade of firms is relatively widespread across all sectors.

To further analyze this, we assess how much of this variability could be related to other observable characteristics correlated to specific sectors, such as firm size. For that we run the following regression

$$Sold_{i} = \sum_{s} \beta_{s} \times Sector_{i,s} + \sum_{q} \beta_{q} \times Size_{i,q} + \sum_{a} \beta_{a} \times Age_{i,a} + \beta_{control} \times X_{control} + \varepsilon_{i}$$
(14)

where $\text{Sector}_{i,s}$ indicates if entrepreneur *i* is in sector *s*, $\text{Size}_{i,q}$ indicates if size of entrepreneurial firm belongs to quartile *q*, $\text{Age}_{i,a}$ if entrepreneur belongs to age group *a*. The dependent variable Sold_i indicates if the entrepreneur sold its business. Figure A.2

exhibit the sector specific effect. We find that, after controlling for these observables, most sectors have a similar propensity. The only sectors with an *unexplained* high propensity to trade are restaurants, hotel and retail sectors, and the ones with low propensities are construction and professional services. These results could be driven by unobservable characteristics such as time-varying demand (restaurants and hotels), fixed costs (construction) and the tradability of the business (professional services).

Sample	Purchased	N(weighted)	N
All owners	-	36,856,133	3,409,393
Respond acquisition	16.0%	20,302,192	2,164,541
Entrepreneurs	17.0%	9,503,681	1,112,254
Employment > 0	25.9%	5,507,460	1,255,134
Receipts > 0	16.9%	$17,\!139,\!950$	1,987,336
Payroll > 0	25.1%	6,045,634	1,338,400
All Size > 0	26.1%	5,344,965	1,216,319
Entrepreneurs + Employment > 0	25.5%	3,167,718	698,651
Manage + Payroll >0	24.7%	3,473,610	745,699
$\text{Share} \ge 50$	13.5%	$16,\!274,\!606$	1,479,855
Share $\geq 50 + \text{Employment} > 0$	23.5%	3,884,071	745,431
Share $\geq 50 + \text{Payroll} > 0$	22.7%	4,320,811	809,769
$\text{Share} \ge 50 + \text{Manage}$	15.4%	8,064,388	827,286
$\text{Share} \ge 50 + \text{Size} > 0 + \text{Manage}$	24.2%	2,385,664	455,442
Weight by Employment	32.2%	$9,\!503,\!681$	1,112,254
Working Age	17.2%	8,298,522	$983,\!598$
Working Age + Employment>0	25.8%	2,838,812	622,336
Hours Worked > 40	19.6%	5,679,652	806,923
Hours Worked $> 40 + $ Employment >0	26.0%	$2,\!545,\!635$	582,966

Table A.4: Share of Entrepreneurs That Purchased Their Business

SOURCE: 2007 SBO.

NOTES: Purchased refers to the percentage of entrepreneurs that acquire its firm through a purchase. Share refers to the normalized entrepreneur's share of the firm. Hours Worked denotes average number of hours per week the owner spends at the firm. Working age are entrepreneurs which are between 24 and 66 years old.

Sample	Owner-level	Firm-level		
		At least one	All	
Respond Acquisition	16.0%	14.7%	12.0%	
Entrepreneurs	17.0%	16.3%	15.0%	
Employment > 0	25.9%	26.8%	20.9%	
Entrepreneurs + Employment>0	25.5%	25.7%	23.2%	
Working Age + Employment>0	25.8%	25.9%	23.7%	
Hours Worked> $40 + \text{Employment} > 0$	26.0%	26.1%	23.8%	

 Table A.5: Share of Firms With Owners That Purchased the Business

SOURCE: 2007 SBO.

NOTES: Hours Worked denotes average number of hours per week the owner spend at the firm. Working age are entrepreneurs which are between 24 and 66 years old.

	Entrepreneurs	+ Employment > 0
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.1%
Franchises only	50.1%	51.8%
Franchise / Total	2.8%	4.8%

 Table A.6:
 Franchises

SOURCE: 2007 SBO.



Figure A.1: Share of Entrepreneurs that Purchased by Sector

SOURCE: 2007 SBO.

NOTES: The rate is constructed as the ratio of firms bought in 2007 to all firms normalized to be 2.5% for employer firms and 2.0% for entrepreneurial firms on average.

Figure A.2: Sector Effect on Probability to Sell a Firm



SOURCE: SBO.

NOTES: Coefficients are normalized to 0 using median of estimates. Standard errors are clustered by sector and state. Units are in percentage points.

Additional to the previous robustness, we analyze the share of entrepreneurs that purchased their firm conditional on other firm observables such as size and age. In our validation exercise, presented in Section 5, we showed that firms *when purchased* tend to be small and young, consistent with the predictions of our model. Nonetheless, the results of this section show that traded firms, *after purchased*, tend to grow bigger and live longer than non-traded firms.

Firm Size. In table A.7 we present the share of entrepreneurs that purchased their business conditional on the size distribution of three different variables representing firm

size: receipts, payroll, and employment. These results show that the trade of firms is even larger, in terms of volume, at the top of the size distribution. For example, in the top 0.1% of receipts, around 39% of entrepreneurs purchased their firm, considerably higher than the unconditional 25.5% share documented for positive employment firms.

Percentile	Variable	Purchased	Average
	Receipts	24.6%	651
Bottom 90	Payroll	24.6%	153
	Employment	25.2%	8
	Receipts	34.6%	8,624
Top 10\Top 1	Payroll	34.4%	1,773
	Employment	37.9%	83
	Receipts	43.8%	57,753
Top 1\Top 0.1	Payroll	40.0%	9,220
	Employment	37.9%	248
	Receipts	39.0%	381,869
Top 0.1	Payroll	35.3%	49,760
	Employment	32.2%	1,374

Table A.7: Firms purchases, by size group

SOURCE: 2007 SBO.

NOTES: Results considering firms with positive employment. Average is computed using both purchased and non-purchased firms. Receipts and Payroll are in thousands ('000) of USD.

Firm Age. Finally, we study the share of traded firms conditional on the age of the firm. Table A.8 shows that that older firms tend to have larger share of trades. This is consistent either with a higher surviving rate of purchased firms, the declining in trade share we observe in the SCF data, or just a higher probability of being purchased for being around more time. Also, this may reflect some life cycle motives since older entrepreneurs probably manage older firms. Related to this, in Appendix A.3.2 we analyze potential life cycle motives for the trade of firms.

A.2.2 Firm Buyers' Previous Occupation - Robustness

Our second main observation is regarding entrepreneurs' previous occupations. In the main text we documented that 62 and 66% of current entrepreneurs have never been self-employed (and hence have never been entrepreneurs) prior acquiring its firm. As a robustness we check how many workers, or not self-employed, transition into entrepreneurship

Firm Age	All Firms	Employer Firms
0-1	8.9%	17.4%
1-2	10.0%	16.3%
2-8	10.9%	16.5%
8-18	13.1%	18.5%
18-28	17.9%	24.9%
+ 28	35.5%	45.2%

 Table A.8: Firms purchases, by age

SOURCE: 2007 SBO.

by acquiring its firm considering alternative definitions of an entrepreneur. In Table A.9 we compute the transition rate from worker to entrepreneur conditional on purchasing the firm for: (i) our baseline definition; (ii) when transition to being the main owner of the firm; (iii) and conditional on large firms. Our results are very similar for all these samples.

 Table A.9: Firm Buyers' Previous Occupation

Sample	Worker Before Purchasing			
Sempre	Baseline	Employment > 0		
Entrepreneur	62.0%	66.0%		
Share> 50	61.9%	64.5%		
Large Firms	69.7%	66.9%		

SOURCE: 2007 SBO.

NOTES: Large Firms as those in the top quintile of the employment distribution.

Type of Purchased Firms. We also analyze whether workers tend to buy certain type of firms. For example, one could argue that worker-buyers concentrate in small non-growth-oriented type of businesses, compared to firms that are acquired by previous firm owners. In Table A.10 we show that there is no clear relation between firm characteristics and the share of firms purchased by workers. If something, this share is slightly larger for older and bigger firms.²⁴

 $^{^{24}}$ The sample is restricted to 2007 such that the characteristics of the firms are approximately to the ones when purchased. For this sample, the share of firm buyers that were workers is slightly lower (less than 60%) than the one of our baseline sample.

	Workers	Purchased
Age of Firm		
0-1	55.0%	59.2%
1-2	54.5%	4.4%
2-8	53.9%	11.3%
8-18	53.4%	9.0%
18-28	57.5%	6.2%
+ 28	58.2%	9.8%
Size of Firm		
Q1	54.2%	22.9%
Q2	54.0%	27.6%
Q3	55.3%	16.4%
$\mathbf{Q4}$	56.3%	22.6%
Q5	58.7%	10.4%

Table A.10: Share of Firm Buyers that were Workers

SOURCE: 2007 SBO.

NOTES: For our calculation we limit to firms purchased in the same year of the survey (2007). To calculate quintiles of size we use employment and exclude non-employer firms. Workers correspond to the ratio of the previously non-self employed entrepreneurs that purchased the firm over the total of firms purchased. Share of purchased firms is the amount of firms purchased by characteristic over all firms purchased.

A.2.3 Trade of Firms Across Time - Robustness

For our third result regarding the decreasing trend in the trade of firms in the SCF we perform similar robustness exercises. Specifically, we consider: (i) our baseline definition, (ii) our second definition with positive employees, (iii) as entrepreneurs may have more than one firm we can count the number of firms purchased, (iii) entrepreneur as main owner (share > 50%), and (iv) baseline definition computed weighting by value of the firm.

Panel (a) of Figure A.3 presents the results for these alternative definitions and samples. Overall, we find that the decreasing trend in the share of business purchased is robust across different definitions, both qualitatively and quantitatively. As in the SBO, we find that there is a level difference between weighting the purchase share by size, positive employment or including all firms. This indicates that larger or more valuable firms are more likely to be traded.

Additionally, we explore whether the decreasing trend in the trade of firms is driven by some specific industry. For this, we perform two robustness exercises: (i) we remove the agricultural sector from our estimates, and (ii) we maintain fixed the share of firms by sector in order exclude changes in the composition of sectors across time. The results are presented in Panel (b) of Figure A.3. We find that qualitatively the decreasing trend is robust to these exercises. However, we find that if we exclude agriculture and fix the share of the sectors to the 1998 shares we have that the decrease in trend remains, but it is reduced to approximately 5 p.p.





SOURCE: 1989-2016 SCF.

NOTES: Fixed 1998 share in panel (b) is created by taking the evolution of purchased firms across time of each sector and aggregate them using their total firm share in 1998.

A.3 Additional Evidence on The Market for Firms

In this section we present some additional results regarding the market for firms. First, we analyze the relation between the number of entrepreneurs, owners, and their equity shares. Second, we analyze the relation between the trade of firms and life cycle motives by analyzing the average age of firm buyers and sellers.

A.3.1 Entrepreneurs, Owners, and Equity Shares

In the main text we reported that in the 2007 SBO more than 80% of entrepreneurs own only one firm. Further we reported that 75% of the private firms have only one entrepreneur while more than 96% of the firms have at most two entrepreneurs. These observations support our assumption that each entrepreneur owns only one firm. In this section we further characterize the number of entrepreneurs and owners per firm and study their equity composition.

The main results of this section can be summarized as follows. On the one hand, we find that in the cases in which a firm has only one entrepreneur the equity tends to be concentrated on the single manager-owner. However, there are also several cases in which the entrepreneur share 50/50 the business equity with another non-manager owner. On the other hand, almost all of the firms with two entrepreneurs tend to share the firm equity 50/50.

We also find a decreasing relation between entrepreneurs' equity shares and the size and age of the firm. Nonetheless, entrepreneurs of firms at the top of the size and firm-age distribution still hold large equity shares on their firms of around 50 to 60%. Lastly, we find that at least 80% of the entrepreneurs have one firm that they manage, and 97% of them have at most 2 firms. Taking all these observations into account we conclude that ownership and management of privately held firms in the U.S. is highly concentrated, and usually in a single entrepreneur.

Number of Owners and Entrepreneurs. Table A.11 reports the share of firms in the 2007 SBO conditional on the number of owners and entrepreneurs. The table shows that 79.8% of the firms have only one entrepreneur, and 97.8% have at most two. For firms with positive employment these numbers are slightly smaller but still are considerably large (73.7 and 96.4%).

		# of Owners or Entrepreneurs			
		1	2	3	≥ 4
All Firms	Owners Entrepreneurs	51.4% 79.8%	$39.3\%\ 18.0\%$	4.5% 1.6%	4.8% 0.1%
+Employment>0	Owners Entrepreneurs	43.0% 73.7%	42.5% 22.7%	7.1% 2.7%	$7.3\%\ 0.1\%$

Table A.11: Share of Firms by Number of Owners and Entrepreneurs

DATA SOURCE: 2007 SBO.

NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

Equity Shares. Figure A.4 shows that for close to 70% of the firms with one entrepreneur this owner holds the 100% equity of the firm. However, for more than 20% of the firms the entrepreneur shares around 50% of the equity with another non-manager owner. On the other hand, in firms of two entrepreneurs the most common arrangement is 50/50 equity shares.

Figure A.4: Equity Shares by Number of Entrepreneurs



SOURCE: 2007 SBO.

Next, we analyze the equity share owned by entrepreneurs conditional on firm size and firm age. Figure A.5 reports that the equity share hold by entrepreneurs slightly drops with firm's size and age. This suggest that larger and older firms do use more equity financing. Nonetheless this negative relation is relatively weak and even for the firms in the top decile of the size distribution around 75% of the firm equity is hold by entrepreneurs. Similar patterns are observed across firms' age distribution. Overall figure shows that

entrepreneurs own, by a wide margin, the largest share of the equity.



Figure A.5: Equity Shares by Firm Size and Age

SOURCE: 2007 SBO.

NOTES: Deciles of size are constructed using the distribution of firms with positive employment. Decile 0 corresponds to firms with zero employees. Values corresponds to the average value of the sum of entrepreneurial ownership share across the firms' size and age distribution.

Number of Firms Owned. Finally, we use data from the SCF to document the number of businesses each entrepreneur owns and manages. Table A.12 shows that at least 80% of the entrepreneurs manage one firm, and less than 3% manage three firms or more. Both, this and the results in the previous part, suggest that ownership and management are very concentrated for privately held firms in the U.S. economy.

	# of managed businesses		
	1	2	≥ 3
All Firms	80.2%	16.9%	2.9%
Employer Firms	82.6%	17.4%	-

 Table A.12: Firms by Entrepreneurs

DATA SOURCE: SCF 1989-2016.

NOTES: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. We use value of the firm as the measure of size.

A.3.2 Life Cycle Motives

Another important motive for the trade of firms, besides financial frictions, are motives related to the entrepreneurs' life cycle. To address this, we study the trade of firms conditional on sellers' age. Panel (a) of Figure A.6 shows that for our baseline definition the trade rate is relative flat. However, it follows a U-shape when we restrict entrepreneurs with employees. This means that the probability of trade is higher for younger and older entrepreneurs. This evidence is consistent with retirement motives playing a role. Additionally, this could also indicate the presence of financial frictions as younger entrepreneurs are more likely to be more financially constrained, compared to middle-age and older entrepreneurs.

The previous result looked at the probability of selling. Another question is about the share of total trades conditioning on the age of the seller. Panel (b) shows that, for both definitions, the share of trades is mostly concentrated among middle-aged entrepreneurs, even though these are the ones that exhibit the lowest trade rates. This result reflects the fact that the age distribution of entrepreneurs also follows an inverted U-shape. Thus, even though old entrepreneurs selling rate is relatively high, the fraction of total trades that could be related to retirement, as proxied by share of sells done by entrepreneurs in the 65+ category, is just around 10%.



Figure A.6: Trade of Firms by Sellers' Age Group

SOURCE: 2007 SBO. NOTES: The trade rates in Panel (a) are normalized to match the total trade rate of 2 and 3%.

A.4 Firms' Trade Rate

We have the stock of traded firms and we want to have an estimate of the annual trade rate. We estimate this rate in two ways, by using the SBO and approximating it with the year 2006 purchased flow corrected by exit and purchasing rate before the survey in 2007, or we estimate it through firms flow equations we are going to explain in this section.

Let us assume the following timing: first firms exit and enter at some rate, and later the purchase happens. Take x stock of firms purchased and y the stock of all firms. Now the flow considering the timing as in the model of purchased firms and total firms are

$$y_{t+1} = y_t \left[1 - \pi_{exit,t}^y + \pi_{entry,t} \right]$$

$$x_{t+1} = x_t \left(1 - \pi_{exit,t}^x \right) + \left[y_{t+1} - x_t \left(1 - \pi_{exit,t}^x \right) \right] \pi_{trade,t_+}$$

where π_{entry} and π_{exit} are the annual entry rate and exit rate, and π_{trade} is the annual rate of firm trade we want to estimate. Using this flow equations, we have that the ratio evolves as

$$\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{ \frac{1 - \pi_{exit,t}^x + \frac{y_t}{x_t} \left[1 - \pi_{exit,t}^y + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi_{exit,t}^x\right) \pi_{trade,t_+}}{1 - \pi_{exit,t}^y + \pi_{entry,t}} \right\}$$

if no growth of the ratio of firms purchased and same exit rate for x and y then

$$\pi_{entry,t} \left[\frac{y_t}{x_t} \left(1 + \pi_{entry,t} - \pi_{exit,t} \right) - \left(1 - \pi_{exit,t} \right) \right]^{-1} = \pi_{trade,t_+} \tag{15}$$

moreover if we assume that entry = exit then

$$\pi_{trade,t_{+}} = \pi_{e,t} \left(\frac{y_t}{x_t} - 1 + \pi_{e,t}\right)^{-1} \tag{16}$$

B Model Appendix

B.1 Additional Derivations

B.1.1 Private firms profit maximization

The solution of entrepreneurs' profit maximization problem, stated in (1), is characterized by the input demand functions

$$k(a, z) = \min\left\{\hat{k}(z), \lambda a\right\}$$
$$l(a, z) = \left[\frac{z\nu}{w}\right]^{\frac{1}{1-\nu}} k(a, z)^{\frac{\theta}{1-\nu}},$$

where \hat{k} is the unconstrained optimal level of capital given by

$$\hat{k}(z) = z^{\frac{1}{1-\theta-\nu}} \left[\frac{\theta}{R}\right]^{\frac{1-\nu}{1-\theta-\nu}} \left[\frac{\nu}{w}\right]^{\frac{\nu}{1-\theta-\nu}}$$

which is only a function of the quality of the entrepreneurial project z.

B.1.2 Public firm optimality conditions

The FOCs of the public firm profit maximization problem are

$$\eta \frac{Y_c}{K_c} = R$$
$$(1 - \eta) \frac{Y_c}{L_c} = w$$

which imply a relation between the public firm capital to output and the equilibrium prices.

B.2 Computational Solution

To solve the model we use projection methods to approximate the value functions $\{V^o, W^o, V^w, W^w\}$. Thus, we need to solve for coefficients $\{g^o_V, g^o_W, g^w_V, g^w_W\}$ such that, at the grid points, satisfy

$$V^{o}(a, z) = \Phi^{z}(a, z)g_{V}^{o}$$
$$W^{o}(a, z) = \Phi^{z}(a, z)g_{W}^{o}$$
$$V^{w}(a, \varepsilon) = \Phi^{\varepsilon}(a, \varepsilon)g_{V}^{w}$$
$$W^{w}(a, \varepsilon) = \Phi^{\varepsilon}(a, \varepsilon)g_{W}^{w}.$$

Note that the FOCs of the public firm give us a relation between K_c/Y_c , w and r. Both K_c and L_c are determined as residuals from the market clearing conditions of capital and labor, thus we can obtain w as a function of r. This considerably simplifies the solution method of our baseline model as we only need to solve for one equilibrium price: r.

B.2.1 Algorithm

The equilibrium objects we need to solve for are

$$\{g_{V}^{o}, g_{W}^{o}, g_{V}^{w}, g_{W}^{w}, n_{dm}^{o}, n_{dm}^{w}, n_{cm}^{o}, n_{cm}^{e}, P_{dm}^{o}, P_{dm}^{w}, P_{cm}^{o}, P_{cm}^{w}, \beta\}$$

where n are the probability densities across states and P are the transition probability matrices (TPMs) across states.²⁵ We solve for these objects using the algorithm now described. In the remaining sections we explain in further detail how some steps of the algorithm are implemented.

Iteration on prices

- 0. Propose an initial guess for r.
- 1. Given r, solve the model (in partial equilibrium).

Iteration on distributions

- 1.0. Propose an initial guess for $\{n_{dm}^o, n_{dm}^w\}$.
- 1.1. Given $\{n_{dm}^o, n_{dm}^w\}$, solve for $\{g_W^o, g_W^w\}$.

Iteration on value functions

- 1.1.0. Propose an initial guess for $\{g_W^o, g_W^w\}$.
- 1.1.1. Solve the DM problem: get $\{g_V^o, g_V^w\}$.
- 1.1.2. Solve the CM problem: obtain e, a' and P_{cm} .
- 1.1.3. Update $\{g_W^o, g_W^w\}$.
- 1.1.4. Iterate $\{g_W^o, g_W^w\}$ until convergence.
- 1.2. Update $\{n_{dm}^{o}, n_{dm}^{w}\}$.
- 1.3. Iterate $\{n^o_{dm}, n^w_{dm}\}$ until convergence.
- 2. Update r such that the capital market clears.²⁶
- 3. Return to 1. until r converges.

²⁵Where $\int n^o(a, z) dadz = s^o$ and $\int n^w(a, \varepsilon) dad\varepsilon = (1 - s^o)$.

 $^{^{26}}$ The labor market always as L_c is equal to the residual between the labor supply and the entrepreneurial sector labor demand.

B.2.2 Computing expectations

Besides the above approximations, it is convenient to approximate the expectations over V^o and V^w such that these are functions of the coefficients g_V^o and g_V^w . Note that when the $(1 - \gamma)$, or the $(1 - \zeta)$, shock hit the expectation over the value of being a firm owner is not a function of previous period z, if any. Thus, we can approximate

$$\mathbb{E}_{z'}\left[V^o\left(a,z'\right)\right] = \Phi^a(a)g_V^{o,E}$$

Then, note that

$$\Phi^{a}(a)g_{V}^{o,E} = \mathbb{E}_{z'}\left[V^{o}(a,z')\right]$$
$$= \sum_{i} \omega_{i}^{z} V^{o}(a,z_{i})$$
$$= \left[\sum_{i} \omega_{i}^{z} \Phi^{z}\left(a,z_{i}\right)\right]g_{V}^{o}$$

where $\{\omega_i\}_i$ are weights that discretize the exogenous process for z'.

This implies that, given g_V^o , the coefficient for the expectation is just

$$g_V^{o,E} = \Phi^a(a)^{-1} \left[\sum_i \omega_i^z \Phi^z(a, z_i) \right] g_V^o$$
$$\equiv \Phi^{z,E} g_V^o$$

and hence

$$\mathbb{E}_{z'}\left[V^o(a, z')\right] = \Phi^a(a)\Phi^{z, E}g_V^z,$$

where $\Phi^{z,E}$ is computed only once.

For the expectation over the value of being a worker at DM we can do similar steps but now accounting for the persistence in ε :

$$\mathbb{E}_{\varepsilon'|\varepsilon}\left[V^{w}\left(a,\varepsilon'\right)\right] = \Phi^{\varepsilon}(a,\varepsilon)g_{V}^{w,E}$$

where a is the policy chosen at CM (i.e. a'), and ε is the current state at CM.

As before

$$\Phi^{\varepsilon}(a,\varepsilon)g_{V}^{w,E} = \mathbb{E}_{\varepsilon'|\varepsilon} \left[V^{w}(a,\varepsilon') \right]$$
$$= \sum_{i} \omega_{i}^{\varepsilon} V^{w}(a,f(\varepsilon,u_{i}))$$
$$= \left[\sum_{i} \omega_{i}^{\varepsilon} \Phi^{\varepsilon} \left(a,f(\varepsilon,u_{i})\right) \right] g_{V}^{w}$$

where $\{\omega_i^{\varepsilon}, u_i\}_i$ are weights and nodes that discretize the exogenous process for ε' . Thus, $\varepsilon'_i = f(\varepsilon, u_i)$, given current period ε .

This implies that, given g_V^w , the coefficient for the expectation is just

$$g_V^{w,E} = \Phi^{\varepsilon}(a,\varepsilon)^{-1} \left[\sum_i \omega_i^{\varepsilon} \Phi^{\varepsilon} \left(a, f(\varepsilon, u_i) \right) \right] g_V^w$$
$$\equiv \Phi^{\varepsilon,E} g_V^w$$

and hence

$$\mathbb{E}_{\varepsilon'|\varepsilon}\left[V^w(a,\varepsilon')\right] = \Phi^{\varepsilon}(a,\varepsilon)\Phi^{\varepsilon,E}g^w_V,$$

where $\Phi^{\varepsilon,E}$ is computed only once.

B.2.3 Solving for g_V^o and g_V^w

Given a $\{n_{dm}^o, n_{dm}^w, g_W^o, g_W^w\}$, we can compute the value at DM for both firm owners and workers. Then we can solve for g_V^o and g_V^w by inverting the basis functions Φ^z and Φ^{ε} .

B.2.4 Solving for a', g_W^o and g_W^w

Having solved for the coefficients g_V^o and g_V^w we can solve the households' problems in the production stage (CM). Given prices, both entrepreneurs and workers problems are a single variable optimization problem in a', which we can solve using golden search or Brent's method.

To obtain g_W^o and g_W^w we use value function iteration. First, by substituting the corresponding optimal policies we obtain two linear systems of equations on g_W^o and g_W^w . Then, we can solve for the coefficients by just inverting the basis functions. For stability reasons we make the update of g_W^o and g_W^w with some dampening.

B.2.5 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n_{dm}^o \\ n_{dm}^w \end{bmatrix}$$
 and $n_{cm} = \begin{bmatrix} n_{cm}^o \\ n_{cm}^w \end{bmatrix}$

where n_{dm}^{o} and n_{cm}^{o} are vectors of size N_{o} and n_{dm}^{w} and n_{cm}^{w} are vectors of size N_{w} . N_{o} and N_{w} are the basis functions grid sizes denoting the number of (a, z) and (a, ε) combinations, respectively. Here $\sum_{i} n_{dm} = 1$, thus, $\sum_{i} n_{dm}^{o} = s_{dm}^{o}$ and $\sum_{i} n_{dm}^{w} = (1 - s_{dm}^{o})$.

Then, the TPMs between DM and CM and CM and DM_{+1} solve

$$(n_{cm})^{\mathsf{T}} = (n_{dm})^{\mathsf{T}} P_{dm}$$
$$(n'_{dm})^{\mathsf{T}} = (n_{cm})^{\mathsf{T}} P_{cm}$$

where $(.)^{\intercal}$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two type of agents:

$$P_{dm} = \begin{bmatrix} P_{dm}^{oo} & P_{dm}^{ow} \\ P_{dm}^{wo} & P_{dm}^{ww} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P_{cm}^{oo} & P_{cm}^{ow} \\ P_{cm}^{wo} & P_{cm}^{ww} \end{bmatrix}$$

where P_{dm}^{oo} captures the transitions of firms' owners that bought another firm or didn't trade, P_{dm}^{ow} is for owners that sold their firm, P_{dm}^{wo} for workers who bought a firm and P_{dm}^{ww} for workers who didn't trade. Regarding CM TPMs, P_{cm}^{oo} is for business owners who operated the firm, P_{cm}^{ow} for owners who didn't operate and went to the labor market, P_{cm}^{wo} for workers who received the $(1 - \zeta)$ shock, P_{cm}^{ww} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n_{dm}^{\mathsf{T}} = n_{dm}^{\mathsf{T}} P_{dm} P_{cm}$$

or

$$\left[I - (P_{dm}P_{cm})^{\mathsf{T}}\right]n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm}P_{cm})^{\mathsf{T}}$ associated with the unit eigenvalue, normalized such that $\sum_{i} n_{dm}(i) = 1$.

C Alternative Models and Parameterizations Appendix

C.1 Employer Firms

This subsection refers to the extension of the baseline model, discussed in Section 7.1, which focus on our second definition of entrepreneurs which requires that the firm has a positive number of employees. According to the 2007 SCF, 5.8% of households are entrepreneurs with employer firms, and earn 19% of total income and hold 33% of total wealth. Besides these differences regarding the role of entrepreneurs in the economy, we also match the higher trade rate of 3% that we documented in the paper empirical sector. We also target the slightly larger share of firms acquired by workers, and equal to 66% for this definition. Our model also does a very good job in matching these alternative set of moments. The table comparing the data targeted moments and ones implied by the model is available upon request.

C.2 Alternative Motives to Trade

C.2.1 Setup

In this version of the model we introduce, in a parsimonious and reduced form way, alternative motives to trade firms that are unrelated to the credit and risk channels arising from financial frictions. Specifically, we assume that upon meeting in the DM subperiod firm sellers might be of two types. With probability $(1-\psi)$ they are *rational sellers* which value their firm as before, and have to be compensated with $\underline{p}(a, z)$ assets. With probability ψ they are *behavioral sellers* which are willing to sell their firm at a discounted price $\kappa \underline{p}(a, z)$, where $\kappa \in (0, 1)$. The behavioral sellers are compensated with a utility transfer which leaves them indifferent between selling the firm or not.

In more detail, the utility transfer aim to capture these alternative motives to sell the firm such as owners' life cycle, or migration, decisions. This utility is similar in spirit to the classical Hicksian compensation such that consumers maintain the same level of utility. Formally, the utility compensation for each behavioral seller with states (a, z) is given by

$$\Xi(a,z): \quad W^w(a+\kappa\underline{p}(a,z),\underline{\varepsilon})+\Xi=W^o(a,z).$$

Under these assumptions the value at the beginning of DM for firm owners is given by

$$V^{o}(a, z) = (1 - \Pr^{o} [\text{buy} | a, z]) W^{o}(a, z)$$
 (no trade or sell)
+ $\alpha_{o} (1 - \psi) \int_{z < \tilde{z}, p < \overline{p}} W^{o}(a - p, \tilde{z}) dN^{o}_{dm}(\tilde{a}, \tilde{z})$ (buy rational seller)

+
$$\alpha_o \psi \int_{z<\tilde{z}, p^b<\bar{p}} W^o(a-p^b, \tilde{z}) dN^o_{dm}(\tilde{a}, \tilde{z})$$
 (buy behavioral seller)

and for workers the value at DM is now

$$V^{w}(a,\varepsilon) = \Pr^{w} [\text{ no trade } | a,\varepsilon] W^{w}(a,\varepsilon) \qquad (\text{no trade}) \\ + \alpha_{w} (1-\psi) \int_{p<\bar{p}} W^{o}(a-p,\tilde{z}) dN^{o}_{dm}(\tilde{a},\tilde{z}) \qquad (\text{buy rational seller}) \\ + \alpha_{w} \psi \int_{p^{b}<\bar{p}} W^{o}(a-p^{b},\tilde{z}) dN^{o}_{dm}(\tilde{a},\tilde{z}) \qquad (\text{buy behavioral seller})$$

where $p^b(a, z) = \kappa p(a, z)$ is the price paid by the buyer when buying from a behavioral seller.

C.2.2 Calibration

To calibrate the model with alternative motives to trade we set the share of behavioral sellers to $\psi = 0.2$ and jointly calibrate the discount parameter κ with the remaining 11 parameters of the model. The number for behavioral sellers is consistent with the share of trades around owners' retirement age that we document in Appendix A.3.2. Besides the 15 moments considered in Section 5, we target two additional moments regarding the frequency of trade across the firm size distribution. Specifically, we target the trade rate of the third and the fifth quintile of the firm size distribution, as given by last period sales. With this procedure we get a value of $\kappa = 0.42$, meaning that behavioral sellers are willing to sell at a 58% discount. Figure C.7 presents the trade rate conditional on firm size in the data and in the two versions of the model. By construction, the model with the alternative motives to trade does a better job in matching the relation between firm size and trade.





C.3 No Public Firms

Usually, it is assumed that large publicly traded firms tend to have sparse ownership and management, but there are notorious cases for which this is not the case. For example, 3 of the top 5 corporations in the Fortune 500 list, in terms of their revenues in 2019, have one owner or a small group of owners that have most of the ownership and voting rights.

In this version of the model, we drop the representative public firm and assume that all the production in the economy is done by entrepreneurial firms which are subject to financial frictions. We adjust our calibration strategy accordingly and target a broader definition of entrepreneurs that only requires households to be business owners. In the 2007 SCF, 14.7% of the households are business owners and they earn 37% of total income and hold 54% of the wealth in the economy. Further we adjust the targeted firms' leverage as the corporate sector features higher leverage, compared to non-corporate firms. Specifically, we target a debt to capital ratio of 0.4. This is consistent with total debt to assets of the private sector as a whole (both corporate and non-corporate) in 2007, according to the Flow of Funds Accounts.

C.4 Alternative Bargaining Power

In our baseline setup we assumed that when a buyer and seller meet the transacted price is determined according to a Nash protocol, stated in (6), in which the buyer has all the bargaining power ($\chi = 1$). In this appendix we analyze how our results change if the trading surplus is distributed differently.

Specifically, we consider the opposite extreme case in which the seller has all the bargaining power ($\chi = 0$). If we assume that $\chi = 0$, the transacted price is equal to

$$p(\tilde{\mathbf{s}}^o, \mathbf{s}) = \bar{p}(\mathbf{s}, \tilde{z})$$

where $\tilde{\mathbf{s}}^o = (\tilde{a}, \tilde{z})$ denotes the states of the seller, $\mathbf{s} \in {\mathbf{s}^o, \mathbf{s}^w}$ the states of the buyer, and \bar{p} denotes the maximum price that the buyer is willing to pay for the firm. Note that different from the case in which $\chi = 1$, the transacted price depends on the states of each specific buyer and on the quality of the firm \tilde{z} to be traded, which complicates the model numerical solution.

To compare these prices against the sellers' minimum price we can compute the average, or expected, transacted price for every seller with a firm quality \tilde{z} when $\chi = 0$ as

$$\mathbb{E}[\bar{p}(\mathbf{s},\tilde{z})] = \alpha_o \int_{z<\tilde{z}, \ \underline{p}<\overline{p}} \bar{p}(a,z,\tilde{z}) \ \mathrm{d}N^o_{dm}(a,z) + \alpha_w \int_{\underline{p}<\overline{p}} \bar{p}(a,\varepsilon,\tilde{z}) \ \mathrm{d}N^w_{dm}(a,\varepsilon)$$
(17)

which depend on both states of the seller, through \underline{p} , the exogenous search frictions (α_o and α_z), and on the endogenous distribution of agents in the market for firms (N_{dm}).

Maintaining the rest of the parameters as in our baseline calibration, on average, the expected buyers' maximum price $\mathbb{E}[\bar{p}(\mathbf{s}, \tilde{z})]$ is 32% larger than the sellers' minimum price $\underline{p}(\tilde{a}, \tilde{z})$. Figure C.8 further analyzes this difference in the transacted prices across the sellers' state space. This panel show that how the trade surplus is divided is particularly important for the owners who have a high firm quality but have few assets to invest. These constrained owners are willing to sell their firms at a relatively low price as it would take them a long time, and high saving rates, to grow out of their borrowing constraints. However, the maximum price that other parties would be willing to pay for the firm is considerably larger than these sellers' minimum price.





NOTES: Difference between the expected buyers' maximum price $\mathbb{E}[\bar{p}(\mathbf{s}, \tilde{z})]$ given by $\chi = 0$, and the sellers' minimum price $p(\tilde{a}, \tilde{z})$ given by $\chi = 1$.

Although not reported in the body of the paper, we also replicate our baseline exercise of shutting down the market for firms under this alternative parameterization. We find a slightly smaller drop of entrepreneurial output of around 4%. This result is explained by the fact that higher prices for the buyers will reduce the remaining resources that new owners have available to invest on the newly acquired firms. Consequently, this reduces the benefits from trading firms in terms of the better allocation of capital and output that these trades can deliver. Despite this, there are no other important qualitative changes relative to our baseline economy.