The Origins of Top Firms*

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Abstract

What are the origins of top firms? What features characterize their life cycle trajectories on the way to the top? Using longitudinal firm-level data, we document novel facts about the first twenty years of the firms that reach the top 1 percent of the size distribution. Compared to the firms in the bottom 99 percent, top firms are eight times larger at entry and grow six times more during their first two decades. In terms of inputs, they start with high capital investments, yet their capital-output ratio and labor share decline as they age. As a result, their profit share is much more backloaded towards the second decade of their life cycle. We show that a firm dynamics model with ex-ante heterogeneity, non-homothetic input costs, and forward-looking financing can explain these empirical patterns. Our quantitative results showcase the importance of accounting for top and bottom firm dynamics for the aggregate implications of financial frictions and recent macroeconomic trends.

Keywords: top 1 percent, firm size distribution, firm dynamics, financial frictions.

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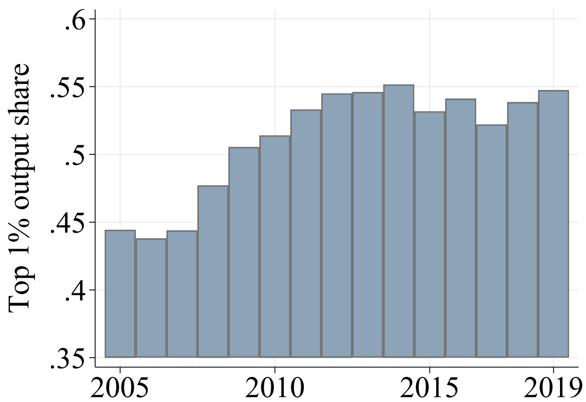
1 Introduction

Top firms, namely the largest in the economy, disproportionately contribute to total economic activity (Axtell, 2001; Luttmer, 2007) and have become increasingly important in recent decades (Kwon, Ma, and Zimmermann, 2024). In Spain, for example, Figure 1 shows that the largest 1 percent of firms accounted for 45% of total output in 2004, rising to 55% by 2019. Recent work argues that the rise of top firms has had significant macroeconomic consequences for the labor share (Autor et al., 2020) and market power (De Loecker, Eeckhout, and Unger, 2020), among other aggregate trends. Yet, despite the importance of top firms for the macroeconomy, little is known about their early years and their way to the top.

Figure 1: Output Share of Top 1 Percent Firms Over Time



Notes: Top 1 percent separately defined for each year controlling for sector fixed effects. Results for Spain using Orbis Historical. Firm-level output is measured as value added, defined as revenue minus a comprehensive measure of non-capital and non-labor costs.

In this paper, we aim to fill this gap by providing novel evidence on the origins and first twenty years of the largest 1 percent of firms (i.e., top firms). We address key questions about the dynamics of top firms: What is their initial size, and how much do they grow? How much capital and labor do they use, and how does input usage vary as they age? How does their profitability evolve over their life cycle? How do these patterns compare to the firms in the bottom 99 percent? Using comprehensive longitudinal firm-level data from several European countries, we find that relative to the bottom 99, firms in the top 1 percent at age 20 start eight times larger and grow six times more in their first two decades. In terms of input usage, they begin with high capital levels, and their capital-output ratio *decreases* by more than 50% over this period. Similarly, their labor share *falls* by roughly 10 percentage points. In contrast, the capital-output ratio among the bottom 99 percent of firms increases over time, and their labor share remains relatively stable.

As a result, top firms' profit share is much more *backloaded* toward the second decade of their life cycle than for bottom firms. These novel findings reveal striking differences in the life cycle trajectories of top and bottom firms.

We show that a parsimonious firm dynamics model with ex-ante heterogeneity, non-homothetic technologies, and forward-looking financing can account for these empirical patterns. In our model, high-growth potential firms have high input-specific fixed costs, creating non-homotheticities in input usage. As firms grow, these costs become small relative to their size, leading to a decline in input-to-output ratios over their life cycle. High-growth potential firms benefit the most from forward-looking financing, which is important to finance input-specific fixed costs. In contrast, low-growth potential firms are more financially constrained and slowly increase their capital-output ratio over time, as in standard models with financial frictions. Our quantitative results showcase the importance of accounting for top and bottom firm dynamics for the aggregate implications of financial frictions and recent macroeconomic trends.

We use Orbis Historical, an extensive firm-level database covering millions of companies worldwide, for our empirical work. We study a sample of European countries for which the data has the best coverage and representativeness (Kalemli-Özcan et al., 2024). Our baseline analysis focuses on Spain, for which Orbis Historical has a long panel and excellent coverage. We present additional results for countries with sufficiently long panel data, such as Belgium, Finland, France, and Sweden. The data includes balance sheets, income statements, key firms' characteristics, such as year of incorporation, detailed industry identifiers, and firm-owner linkages.

We define *top* firms as those in the top 1 percent of the output distribution and *bottom* firms as those in the remaining 99 percent. For our longitudinal analysis, firms are classified as top or bottom based on their size at age 20.[1] Hence, we concentrate on firms that survive to age 20 and track their trajectories back in time. Using an empirical model with year and firm fixed effects, we estimate top and bottom firms' life cycle trajectories throughout their first twenty years. Our key empirical findings are as follows.

First, we document stark differences in the output trajectory of top and bottom firms. Importantly, these differences are present in their initial size and subsequent growth. Firms that make it to the top 1 percent at age 20 are 7.1 times larger than the average entrant at age 0-2, and they grow by a factor of 10.6 times during their first two decades. Firms in the bottom 99 percent at age 20 have an initial output of 0.87 times the average entrant, growing 1.7 times during their first 20 years. Hence, top firms at

[1] As the size distribution spans out as firms age, it is worth noting that a firm in the top 1 percent at age 20 also belongs to the top 1 percent of the entire firm size distribution.

age 20 are 8.2 times larger in their early years and grow 6.3 times faster than bottom firms. These output dynamics are consistent with the view that firms are ex-ante heterogeneous, both in their initial size and growth profiles (Sterk, Sedláček, and Pugsley, 2021).

Next, we study how top and bottom firms use capital and labor over their first twenty years. We uncover a key novel fact: top firms have high initial input usage, particularly capital, and their input-to-output ratios significantly fall over their life cycle. In contrast, bottom firms exhibit either increasing or stable input-to-output ratios. In detail, the top 1 percent of firms at age 20 enter with a high capital-output ratio, approximately 4.8, and it sharply *declines* by around one-half in their first two decades. In contrast, firms at the bottom 99 by age 20 start operating with a low capital-output ratio, around 2.4, and it *increases* by roughly 20% in their first twenty years. Regarding labor usage, we find that the labor share—defined as labor costs over output—is initially similar, around 0.7, early in the life cycle of top and bottom firms. Yet, the evolution of top and bottom firms' labor share is notably different. While the labor share of the bottom 99 percent firms is roughly *constant*, firms in the top 1 percent at age 20 exhibit a *fall* in the labor share of 11 percentage points in the first two decades of their life cycle.

These significant differences in top and bottom firms' input usage have large implications for the path of profits. Indeed, we document that the profit share, defined as profits over output, of the top 1 percent at age 20 is much more *backloded* towards the second decade of their life cycle than for bottom firms. These differences in profitability are significant. The profit share of firms in the top 1 percent at age 20 dramatically increases from roughly 0 to 0.25 in their first two decades. In contrast, the profit share of firms in the age 20 bottom 99 barely increases in twenty years, from 0.09 to 0.12.

We show that our key novel facts about input usage and profitability over the life cycle of top and bottom firms remain robust across multiple checks. Moreover, we extend our analysis to other points of the firm size distribution beyond the top 1 and bottom 99, specific production sectors, and additional countries beyond Spain, finding consistent results. In addition to our core findings, we document the dynamics of debt and equity financing for top and bottom firms, which shed light on how firms finance their inputs and growth. We find that relative to firms in the bottom 99 percent, top firms borrow more, deleverage less as they age, and rely more on equity financing in their early years.

Having documented strikingly different trajectories over the first twenty years of top and bottom firms, we conclude our empirical analysis by presenting some suggestive evidence on the sources of firm ex-ante heterogeneity. Using firm-owner linkages, we present a taxonomy based on the characteristics of firms' owners. We find that firms in the top

1 percent at age 20 are nearly 11 times more likely to be foreign-owned than firms in the bottom 99 percent. Further, they are almost 3 times more likely to have owners with multiple firms, underscoring the role of business groups and serial entrepreneurs.

In the second part of the paper, we show that a general equilibrium firm dynamics model with ex-ante heterogeneity, non-homotheticities in production, and forward-looking financing can account for our novel empirical findings. We assume that firms are *ex-ante heterogeneous* in their initial productivity, growth profiles, and technology types, which determine their span of control, input shares, and non-homotheticities. Technologies are scarce as, in every period, the number of technology-specific potential entrants is fixed. The distribution across technology types shapes the economy's technological frontier, and has first order implications for total output, the labor and profit share, and concentration.

In the spirit of the classical Stone-Geary preferences (Geary, 1950; Stone, 1954), we introduce *non-homotheticities* in production by assuming a minimum level of capital and labor for firms to operate. Thus, it is worth noting that our modeling of non-homotheticities is equivalent to assuming input-specific fixed costs. While we model these non-homothetic fixed costs in reduced form, we show that they can be microfounded through input indivisibilities or complementarities between inputs and productivity.

Firms have access to debt and equity *financing*. In terms of debt, besides the standard asset-based backward-looking component, we introduce a value-based forward-looking term, which jointly determine firms' borrowing limits. The forward-looking component allows firms with higher growth potential to borrow more than those with more modest potential, even if their current earnings are similar. Additionally, we incorporate costly equity injections as a second form of forward-looking financing.

Our theory provides a parsimonious explanation for the patterns observed in the data. A central feature of our model is that top firms with high-growth potential face large input-specific fixed costs. Firms can partly finance these costs through forward-looking financing. As firms grow, the non-homothetic fixed costs become smaller relative to output, reducing the capital-output ratio and labor share over the life cycle of top firms. Notably, these dynamics in capital and labor lead to a strongly backloaded path for the profit share, initially close to zero but rising significantly as firms age. In contrast, bottom firms with low-growth potential initially face tighter financial constraints as they benefit less from forward-looking financing. As they accumulate assets and relax their borrowing constraints, their capital-output ratio increases. As financial constraints do not distort labor, the labor share remains flat through their life cycle. Thus, our model captures the empirical patterns through high growth and non-homothetic input costs for the top

1 percent and financial frictions for the bottom 99 percent.

We examine alternative explanations for the life cycle trajectories of the top firms, such as product and labor market power and input-biased productivity growth. However, we argue that these alternative views struggle to jointly explain the input-to-output patterns and the capital-labor composition.

Following the firm dynamics literature, we calibrate our model to match firm growth, exit rates, and output concentration in the data. Unlike previous work, we directly target moments related to our novel findings on the life cycle growth, input usage, profit share, and debt and equity financing of top and bottom firms. In our quantification, we use two types of technologies: top and bottom.[2] The degree of ex-ante heterogeneity is informed by the initial size and growth patterns of top and bottom firms. Our model does a good job of quantitatively capturing the first twenty years of top and bottom firms.

We conduct four exercises in our model economy to study whether the life cycle dynamics of top firms are relevant at the aggregate level. The first two focus on the macroeconomic importance of input-specific fixed costs and forward-looking financing, two key factors in the life cycle of top firms. Motivated by recent aggregate trends, the last two exercises examine the aggregate implications of growth driven by technological composition improvements (e.g., greater diffusion of ideas and lower adoption costs) and the effects of long-run changes in interest rates, highlighting the importance of accounting for top firms' dynamics. The results are as follows.

In our first exercise, we quantify the aggregate importance of input non-homotheticities. First, we ask: how large are these input-specific fixed costs in terms of total output? We find that, in our calibrated model, the total input-specific fixed costs are only 0.3% and 0.2% of aggregate output, for capital and labor, respectively. While these costs appear negligible in the aggregate, we show that these non-homotheticities, because of their significant role early in the life cycle of top firms, can have very sizable aggregate implications. We assess their indirect relevance by removing these non-homotheticities from the model and solving for the new steady state. We find that after eliminating capital-specific fixed costs, aggregate output increases by almost 70%, an extremely large change. This significant indirect effect is explained by changes in top technologies' entry and exit margins, which become more prevalent in the absence of capital non-homotheticities. We find almost no aggregate indirect effects after eliminating labor-specific fixed costs.

Second, we explore the role of equity and forward-looking debt financing. We first remove firms' ability to issue equity while allowing them to continue borrowing against their

[2]Firms with top technologies do not necessarily map one-to-one to the top 1 percent of the size distribution as some bottom technology firms might reach the top by size due to high ex-post shocks.

assets and future profits. This results in an aggregate output decline of 7%, driven by a lower share of top technologies in the economy. Similarly, removing only forward-looking debt financing results in a 6% decline, again due to a weaker technological composition. Finally, eliminating both sources of forward-looking financing compounds the output drop to 19%, driven not only by a lower share of top technologies but also by the lack of firm entry despite significantly lower equilibrium wages. These output losses are nearly twice as large as those caused by the absence of traditional backward-looking financing.

Overall, these findings underscore the critical role of forward-looking financing for top firms, which operate with a backloaded profit structure due to high initial input-specific fixed costs. By enabling these firms to finance high initial input usage and endure periods of low profitability, forward-looking financing reduces premature exits and facilitates the entry of top technologies, with significant aggregate implications.

In our third exercise, we use the model as a laboratory to study the effects of aggregate growth driven by technological composition improvements. In detail, we analyze changes in the economy's ex-ante technological composition and explore the implications for the aggregate labor and profit shares and firm concentration in the long run (comparing steady states). Our model indicates that if economic growth is primarily driven by a larger share of potential entrants with top technologies, the labor share declines, while the profit share and firm concentration increase. These shifts can be quantitatively large, even for relatively small changes in the technological frontier.

Finally, motivated by the secular changes in the interest rate over the past decades, we explore whether the presence of top and bottom firms amplifies or dampens the long-run effects of interest rate changes. To do so, we compare two models: our baseline economy that accounts for top and bottom firm dynamics and a counterfactual model calibrated to match analogous life cycle moments for the average firm. We find that the aggregate effects of interest rate changes are significantly *amplified* in our baseline economy, where capital-specific fixed costs of top firms are directly affected by changes in the interest rate. This result highlights the importance of accounting for the origins and trajectories of top firms for macroeconomic outcomes.

Related Literature Our paper contributes to the following strands of literature.

Top Firms in Macroeconomics. First, our paper contributes to the literature on the role of top firms in macroeconomics. One strand of this literature studies the relationship between business cycles and top firms. A substantial body of work (e.g., Gertler and Gilchrist, 1994; Crouzet and Mehrotra, 2020) documents how the cyclical properties of large firms differ from those of small firms, while other research (e.g., Gabaix, 2011; Giovanni, Levchenko, and Mejean, 2024) investigates the extent to which shocks to top firms influence aggregate fluctuations. Another strand of literature focuses on the long-run implications of the rise of top firms.[3] Existing papers in this literature have studied the relationship between the rise of top firms and market power (e.g., Hall, 2018; Van Reenen, 2018; De Loecker, Eeckhout, and Unger, 2020; Covarrubias, Gutiérrez, and Philippon, 2020), the decline in the labor share (e.g., Autor et al., 2020; Kehrig and Vincent, 2021), and their role in innovation (e.g., Garcia-Macia, Hsieh, and Klenow, 2019; Braguinsky et al., 2023; Ayyagari, Demirgüç-Kunt, and Maksimovic, 2023; Casal, 2025).[4]

Our paper contributes to this literature by documenting novel facts about the first twenty years of the largest 1 percent firms. Hence, unlike most previous work in this literature, we are not only interested in top firms once they are large and mature but also in their early years and their way to the top. In this line, our paper relates to Luttmer (2011) that documents the employment histories of U.S. firms with more than 10,000 employees in 2008 and interprets the relatively young age of large firms through a firm dynamics model with ex-ante heterogeneity and ex-post growth. Perhaps closest to our work is Ma et al. (2025), which documents the birth rates of the largest companies in the U.S. over time. It finds evidence of special cohorts of entrants in specific sectors, leading to significant shifts in the age distribution of top firms. As in those papers, we also track top firms backward and study their origins and life cycle growth. However, unlike previous work, we document input usage, profits, and financing of the largest 1 percent firms and contrast their trajectories to those in the bottom 99 percent. In addition to our empirical contribution, we argue that these patterns provide key insights into top firms' dynamics and that accounting for them has important macroeconomic implications.[5]

Life Cycle of Firms. Our paper also relates and contributes to the literature that empirically and theoretically studies the life cycle of firms (e.g., Davis, Haltiwanger, and Schuh, 1996; Haltiwanger, Jarmin, and Miranda, 2013; Hsieh and Klenow, 2014; Kochen, 2023; Dinlersoz et al., 2024; Eslava, Haltiwanger, and Urdaneta, 2024). Most closely related to

[3]Corporate concentration has increased in the U.S. (Kwon, Ma, and Zimmermann, 2024) and the rest of the world (Ma, Zhang, and Zimmermann, 2025) over the last 100 years. Similar trends have been documented for Europe (Bajgar et al., 2023; Koltay, Lorincz, and Valletti, 2023) and Korea (Choi et al., 2025). Other work highlighted divergences between local and national concentration (Rossi-Hansberg, Sarte, and Trachter, 2021) and between product and market concentration (Amiti and Heise, 2025).

[4]The mechanisms driving the rise of top firms are contested and intertwined, with explanations ranging from changes in firms' technology and economies of scale (Aghion et al., 2023; Hsieh and Rossi-Hansberg, 2023; Lashkari, Bauer, and Boussard, 2024; Hubmer and Restrepo, 2022; Firooz, Liu, and Wang, 2025, among others) demographic changes (Hopenhayn, Neira, and Singhania, 2022; Karahan, Pugsley, and Şahin, 2024; Peters and Walsh, 2024), antitrust and regulation (e.g., Philippon, 2019; Akcigit and Ates, 2023), or declining interest rates (Kroen et al., 2022; Chatterjee and Eyigungor, 2023).

[5]Our empirical analysis focuses on Europe, for which we have longitudinal data on balance sheets, income statements, and ownership data. Comparable data is not available for the U.S. beyond specific sectors (e.g., manufacturing, which represents only 10% of GDP) or publicly listed firms.

our empirical work within this literature is De Haas, Sterk, and Van Horen (2022), which clusters firms based on their initial observable characteristics and studies them *forward*.[6] They find persistent differences across clusters. Our two papers complement each other, as our paper identifies the largest firms at age 20 and tracks them *backward*.[7]

From a theoretical perspective, there has been significant interest in the literature on how backloaded profits are over the life cycle of firms. For example, Atkeson and Kehoe (2005) develops a theory of the life cycle of organization rents where interest rates and backloaded rents play a crucial role. Cole, Greenwood, and Sanchez (2016) develops a model where financial development determines technology adoption when the most promising technologies have backloaded returns. More recently, Van Vlokhoven (2021) studies the aggregate implications of profits becoming more backloaded over the life cycle of firms. Our paper contributes to this literature by providing novel evidence on the path of profits for firms at different points of the size distribution, delivering new insights for modeling and theories on the life cycle of firms. To the best of our knowledge, our paper is the first to document a strongly backloaded profit share for the largest 1 percent of firms and a relatively flat life cycle pattern for the bottom 99 percent.

Firm Dynamics. Lastly, our paper contributes to the vast literature on firm dynamics (e.g., Jovanovic, 1982; Hopenhayn, 1992; Hopenhayn and Rogerson, 1993; Melitz, 2003; Luttmer, 2007). Closest to our work is Sterk, Sedláček, and Pugsley (2021), which using insights from the earnings dynamics literature, estimates the importance of ex-ante heterogeneity for firm growth and the firm size distribution. Some examples of more recent and contemporaneously developed work include Hubmer et al. (2024), which analyzes the role of productivity and returns to scale in shaping the revenue distribution, and Mertens and Schoefer (2024) that study firms' input substitution in the short and medium term.

Our contribution to this literature is twofold. On the empirical side, we document novel facts about growth, input usage, and profits over firms' life cycle and uncover strikingly different trajectories for firms at the top and bottom of the size distribution. On the quantitative side, we show that in addition to ex-ante heterogeneity in productivity profiles, which is crucial for capturing the growth patterns of top and bottom firms, non-homotheticities in production are essential for explaining the input usage and profitability of top firms in the data. We also quantify the importance of forward-looking financing

[6]Within the entrepreneurial literature, Guzman and Stern (2015) follows a similar approach using information near the firm foundation to identify the best predictors of firm growth. In Appendix A.3, we perform a similar analysis in our data and find that firms with a large initial size, higher capital-output ratios, and more frequent equity injections early in their life cycle are more likely to reach the top.

[7]From a methodological standpoint, our backward-looking approach resembles that of Halvorsen et al. (2024), which studies the life cycle dynamics of the wealthiest households using Norwegian panel data.

in this setup, where high-growth technologies require large input-specific investments and have backloaded returns. We show that forward-looking financing is twice as important as standard backward-looking collateral constraints under top and bottom firm dynamics.[8]

Outline The rest of the paper is organized as follows: Section 2 describe our data and presents descriptive statistics; Section 3 presents our key empirical findings on the origins and life cycle of top firms; Section 4 presents the model; Section 5 describes the predictions of input usage in data and theory; Section 6 presents our calibration strategy; Section 7 presents our aggregate results; and finally, Section 8 concludes.

2 Data and Descriptive Statistics

This section describes the data used in the paper. It also describes the sample selection and provides definitions of the main variables used in the analysis. Furthermore, it presents some motivating cross-sectional descriptive statistics.

2.1 Orbis Historical

To document the origins of the top firms, we use Orbis Historical, an extensive firm-level database covering millions of companies worldwide. Moody's Bureau van Dijk (BvD) compiles this database by aggregating data from various sources, such as national business registries, and harmonizes it into a globally comparable format. We focus on a sample of European countries with the best coverage in Orbis (Kalemli-Özcan et al., 2024). For our baseline results, we focus on Spain, where Orbis Historical has a long panel of 28 years, between 1992 and 2019, and has excellent coverage, especially from 2000 onward. Figure A.1 in the Appendix shows that Orbis Historical covers, on average across years, 78% of aggregate revenue in Spain, with higher coverage for the most recent years. Furthermore, Figure A.2 shows that the data captures well the firm size distribution reported in official statistics. In addition to our baseline analysis for Spain, we present results for other European countries with sufficiently long panel data in Orbis, such as Belgium, Finland, France, and Sweden.

Sample Selection Our analysis focuses on the non-financial private sector.[9] Throughout the paper, we define firms using companies' unconsolidated accounts. We restrict to the firm-year observations with available data for the wage bill, output, capital, debt, net

[8]Some early work studying non-homotheticicities in production inputs include Lau and Tamura (1972), Hanoch (1975), and Sato (1975). More recently, Lashkari, Bauer, and Boussard (2024) and Eckert, Ganapati, and Walsh (2024) study production non-homotheticities focusing on ICT capital. On the other hand, forward-looking financing in the form of debt and equity has been studied in the context of quantitative firm dynamics models in Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Hennessy and Whited (2007), and Brooks and Dovis (2020).

[9]In detail, we exclude firms in the following sectors: Financial and Insurance (K), Public Administration and Defense (O), Activities of Households as Employers (T), and Extraterritorial Organizations and Undifferentiated Goods and Services (U).

worth, and profits. We exclude the year and 4-digit NACE sectors with fewer than 100 observations to adequately control for time and sector fixed effects. After applying this sample selection criterion, we have a sample for Spain comprised of 9,782,934 firm-year observations from 1,294,582 firms. For our longitudinal results, we further restrict the analysis to a sample of firms we observe at age 20 and can track backward for at least 10 years, resulting in 1,997,695 firm-year observations from 138,113 firms.

Variable Definitions We study the evolution of firm-level output, wage bill, capital, and profits. We convert all nominal variables to real terms using country-specific CPI deflators. We measure output using value-added, defined as revenue minus a comprehensive measure of costs, which excludes labor expenses and capital depreciation. For the wage bill, we use all labor costs. Our baseline measure of capital is equity plus net financial debt. We also provide results measuring capital as tangible assets, as well as tangible plus intangible assets. For firms' profits, we use a model consistent definition of economic profits obtained after setting values for the capital depreciation and the interest rate. We show that our results are similar if we use the variable net income reported in the data. Appendix A.1 defines these variables and provides further measurement details.

Central to our analysis of the life cycle trajectories of top firms is the measurement of firm age, which we calculate using the year of incorporation reported in the data. In some cases, the incorporation year in Orbis and the actual foundation year might differ because of changes in the company's legal name due to restructures or mergers and acquisitions (M&A). To account for this issue, which primarily concerns the largest corporations, we searched for the foundation year of all the top 1 percent firms at age 20 in our sample. Whenever possible, we retrieve this information from the company website. Otherwise, we use publicly available online information. If the incorporation and foundation year differ, we use the foundation year of the company to compute the firm's age. For the case of foreign multinationals, we measure firm age since entering the Spanish market.

Firm-Owner Linkages In addition to our longitudinal analysis, we present a taxonomy for top and bottom firms given the characteristics of their owners. For that, we use firm-owner linkages in Orbis Historical, available from 2007 onward, which include annual records with owners' names and equity shares. The data also specifies whether the owner is an individual, another company, a financial institution, or a government. Given firms' sometimes complex ownership structures, we identify firms' ultimate owners by sequentially matching the ownership files as in Peter (2021) and Kochen (2024). The main objective of this iterative procedure is to account for the cases where other companies own firms by assigning the respective ownership shares of the parent company.[10] Indeed, for the sample of Spanish firms we study, 20% of the direct owners are other companies. For our baseline results, we focus on the ultimate owner obtained after three rounds of matching the ownership files. As we elaborate below, solely studying direct owner linkages would underestimate the importance of foreign and multiple firm owners.

2.2 Top Firms

We define top firms as those in the top 1 percent of the output distribution. To account for variation in firm size by year and sector, we use a residualized measure of output when making this classification, which we obtained after controlling for year and NACE 4-digit industry fixed effects. We rank firms across the entire distribution and conditional on age. Thus, we refer to a firm as being in the *top 1 percent at age a* if its output at age a is in the top 1 percent of the output distribution among all firms of age a observed in the data. We use this age-based definition of top firms in our life cycle analysis to track backwards the origins of the top and bottom firms at a given age a.

Our longitudinal analysis studies the origins of the top 1 and bottom 99 percent of firms at age 20. We focus on the first twenty years of a firm's life cycle, which is the largest horizon we can track firms backward for at least 10 years without running into small sample concerns. Thus, given our sample period, our longitudinal analysis conditional on size at age 20 focuses on the firms born between 1980 and 1998. As we document below, firms in the top 1 percent at age 20 are also in the top 1 percent of the full-size distribution, reflecting both firm growth over the life cycle and survival bias. Yet, many top firms are older than 20 years, as Figure A.5 in the Appendix shows. While we do not observe the origins of older cohorts in our data, it is worth noting that the top firm status at age 20 is highly persistent. Indeed, Appendix A.2 documents that top firms at age 20 are likely to remain at the top in subsequent years. This evidence suggests that the life cycle trajectories of the top firms at age 20 that we document for the 1980 to 1998 cohorts can provide valuable insights into the trajectories of both older and younger cohorts.

2.3 Cross-Sectional Descriptive Statistics

Before presenting our longitudinal analysis, we characterize top firms in the cross-section. We report standard income and balance sheet variables and key ratios we study in the longitudinal analysis between top and non-top firms and document top firms' production sectors and business structures. Overall, the cross-sectional patterns align with previous studies. Yet, it is worth noting that many top firms are privately held and

[10] Hence, for example, if 100% of company A is owned by company B, the procedure assigns the ownership information of B to A. If x% of company A is owned by company B, only x% of the ownership of B will be transferred to company A.

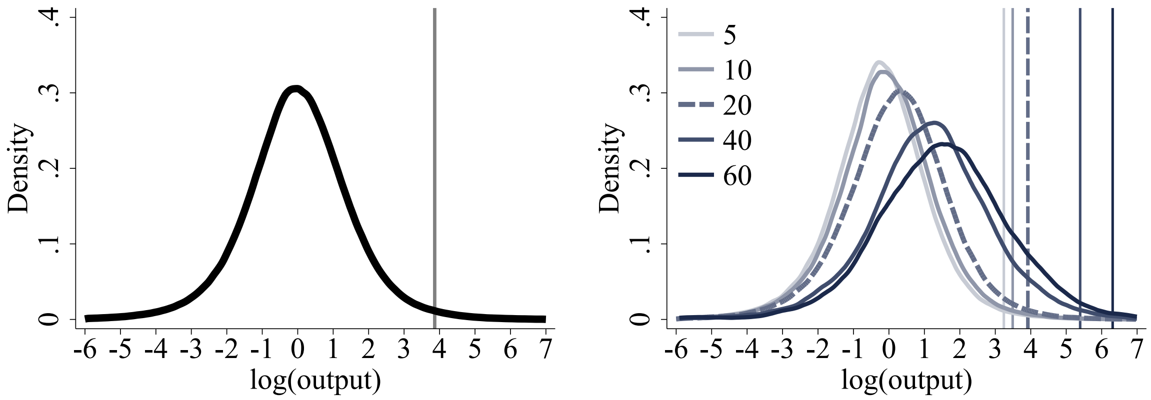
are not concentrated in a specific production sector, such as manufacturing.

Output Distribution Panel (a) of Figure 2 shows the distribution of log output for the whole sample of firm-year observations in Spain. The vertical line indicates the 99th percentile. Thus, the top 1 percent of firms we primarily study are those above this threshold. Panel (a) shows the extreme dispersion in the empirical distribution of firm size. To put this in perspective, the 99th percentile of log output is 3.86, which implies that a firm in the 99th percentile has 47.7 times higher output than the average firm. Alternatively, we can measure the share of production accounted for by the largest firms. Figure 1 shows that, on average, the largest 1 percent of firms account for more than 50% of total output. The high concentration of production by the top firms is a well-known fact, which has been documented for both the U.S. (Axtell, 2001; Luttmer, 2007) and Europe (Bajgar et al., 2023; Koltay, Lorincz, and Valletti, 2023).

Panel (b) presents the log output distribution conditional on firms' age for five age levels: 5, 10, 20, 40, and 60. The vertical lines represent the 99th percentile of each distribution. The output distribution by age shows that firms grow over their life cycle and that the size distribution spans out as firms get older, as documented by Cabral and Mata (2003) and Sterk, Sedláček, and Pugsley (2021) using Portuguese and U.S. data, respectively. It is worth noting that the 99th percentile of the log output for age 20 firms is 3.92 (dashed vertical line), which is higher than the 99th percentile of the whole distribution. Hence, the top 1 percent of firms at age 20, studied in our longitudinal analysis below, also belong to the top 1 percent of the entire firm size distribution.

Figure 2: Output Distribution

(a) All **(b)** By Age



Notes: Residualized log output distributions after controlling for sector and year fixed effects. Log output is normalized to zero using the mean of the unconditional distribution. Output is measured as value added using a comprehensive measure of non-labor and non-capital costs. Distributions are approximated by a kernel density. The vertical line corresponds to the 99th percentile of the distribution. Panel (a) shows the distribution for all the firms and panel (b) shows the distribution conditional on firm age.

Top 1 vs Bottom 99: Size and Main Variables The top panel of Table 1 presents summary statistics for top and bottom firms, defined using the whole sample. All the ratios reported in the table are weighted by their respective variables in the denominator. Hence, we can interpret them as the ratio for the representative firm in each group.[11] On average, considering output or sales, firms in the top 1 percent of the size distribution are 90 times larger than the average firm in the bottom 99 percent. Top firms' have a higher average capital-output ratio k/y, equal to 3.4, while this number is 2.7 for the bottom firms. Notably, firms in the top 1 percent have an average labor share, defined as labor costs over output wl/y, of 0.51, 17 p.p. lower than those in the bottom 99 percent, equal to 0.68. This fact is consistent with the evidence in Autor et al. (2017), which documents that superstar firms, the largest and most productive in a given industry, have a low labor share of value added. A perhaps more novel fact is that the profit share, defined as profits over output π/y, of the top 1 percent firms is two times higher than that of the bottom 99 firms: 0.27 and 0.13, respectively. Top firms also have higher leverage, defined as net financial debt over capital b/k, equal to 0.39 relative to 0.31 for the bottom firms.

Top 1 vs Bottom 99: Production Sectors and Legal Status The bottom panel of Table 1 documents the composition of top and bottom firms across industries and by their legal status. We report the share of observations, output, and capital across these categories, conditional on top and bottom status. Importantly, our data allows us to study all economic sectors, not only manufacturing, and includes public and private firms. By construction, our definition of top firms accounts for average differences across sectors, partly explaining the similarities in the sectoral composition of top and bottom firms. Yet, top firms have a higher share of observations and output in services. There are also significant differences in firms' legal status. Among the top 1 percent firms, 3% are publicly listed in the stock market, 53% are public companies (analogous to C-corporations in the U.S.), and 44% are private companies. In contrast, the bulk of the firms in the bottom 99 percent, 86% of them, are private.

To summarize, the largest 1 percent of firms in the cross-section exhibit a higher capital-output ratio, lower labor share, higher profit share, and are more leveraged. They are also more likely to operate in the service sector, with over half being public. In the next section, we exploit the longitudinal dimension of our data to study the origins and life cycle trajectories of top firms at age 20.

[11] The average weighted by the denominator is equal to the ratio of the aggregate amounts, which we can interpret as the ratio for the representative firm. For example, the weighted average capital-output ratio is equal to: $\sum_i \frac{y_{it}}{Y_t}\frac{k_{it}}{y_{it}} = \frac{K_t}{Y_t}$, where $Y_t = \sum_i y_{it}$ and $K_t = \sum_i k_{it}$. Note that this measure is less sensitive to outliers than the simple average of the ratios.

Table 1: Summary Statistics

	Bottom 99 Percent			Top 1 Percent		
	Mean	Median	SD	Mean	Median	SD
Age	12.5	11.0	9.6	23.0	20.0	15.9
Output (USD, millions)	0.5	0.2	1.7	46.1	15.1	204.9
Sales (USD, millions)	2.1	0.4	13.6	181.6	42.0	829.6
Employment	12.4	5.0	82.1	590.8	196.0	2294.9
Capital-output ratio	2.7	1.3	6.7	3.4	1.6	6.2
Labor share	0.68	0.71	0.28	0.51	0.54	0.32
Profit share	0.13	0.13	0.34	0.27	0.26	0.38
Leverage	0.31	0.30	0.44	0.39	0.44	0.36
	Share			Share		
	Obs.	Output	Capital	Obs.	Output	Capital
Manufacturing	0.32	0.44	0.38	0.27	0.30	0.29
Retail and Transport	0.29	0.25	0.19	0.24	0.24	0.16
Services	0.35	0.26	0.35	0.42	0.33	0.31
Other	0.05	0.05	0.07	0.08	0.13	0.24
Listed Firms	0.00	0.00	0.02	0.03	0.15	0.32
Public Firms	0.14	0.40	0.38	0.53	0.62	0.46
Private Firms	0.86	0.59	0.60	0.44	0.23	0.22

Notes: Top 1 and Bottom 99 Percent defined by output, as described in Section 2.1. Variables in dollars are in 2015 USD using constant prices at constant exchange rates. The capital-output ratio, the labor share (labor costs over output), and the profit share (profits over output) are weighted by output. Sectors are based on NACE 2-digit classifications: Manufacturing is $\texttt{nace} \in [10, 33]$ and includes Construction $\texttt{nace} \in [41, 43]$, Retail and Transport is $\texttt{nace} \in [45, 53]$, and Services excluding Retail and Transport is $\texttt{nace} \in [55, 82]$, $\texttt{nace} \in [85, 88]$, and $\texttt{nace} \in [90, 96]$. Other corresponds to those sectors not listed before. Listed Firms are those publicly listed throughout our sample. Public Firms are public limited companies (*Sociedades Anónimas*) and Private Firms are partnerships and private limited companies (*Sociedad de Responsabilidad Limitada*). See Appendix A.1 for definitions and measurement of these variables.

3 The Origins of Top Firms

This section documents how the top 1 percent of firms start and how they grow. Our analysis focuses on the evolution of firm size and key ratios characterizing firms' use of input and profits. We first describe our empirical strategy and present the empirical specification we use to estimate the life cycle of firms throughout their first two decades. Second, we present our main results for the top 1 and bottom 99 percent of firms. We present additional results along the firm size distribution and results for other countries.

3.1 Empirical Specification

We study the origins of firms conditional on their size at age 20. Thus, our analysis focuses on the firms that survived at least to that age. A way to think about our empirical strategy is that we study firms' life cycle *backward*. Given their position in the age 20 size distribution, we trace their trajectories back in time to when they started operating.

We begin by using our residualized measure of output, which controls for average sector and year differences, to classify firms with available data at age 20 into two groups: the top 1 percent and the bottom 99 percent. We then focus on firms observed for at least 10 years between ages 0 and 20 to estimate firm fixed effects more precisely.[12]

Having classified firms, we study the first twenty years of firms' life cycle by estimating the following non-parametric specification for different dependent variables x_{it}

$$x_{it} = \sum_{a \in \mathcal{A}} \beta_a^{\mathcal{T}} D_{it}^a \times \mathcal{T}_i + \sum_{a \in \mathcal{A}} \beta_a^{\mathcal{B}} D_{it}^a \times \mathcal{B}_i + \sum_{a \in \mathcal{A}} \beta_a^{\mathcal{R}} D_{it}^a \times \mathcal{R}_i + \alpha_i + \alpha_t + \varepsilon_{it} \tag{1}$$

where D_{it}^a is a dichotomous variable equal to 1 if firm i belongs to age group a at period t. The set $\mathcal{A}$ includes ten age groups: age 0-4, 5-6, 7-8, 9-10, 11-12, 13-14, 15-16, 17-18, and age greater than or equal to 21. Hence, the omitted group, which will serve as a reference, is 19-20. The variable $\mathcal{T}_i$ equals 1 if firm i is in the top 1 percent of the output distribution at age 20, and 0 otherwise. Analogously, $\mathcal{B}_i$ equals 1 if firm i is in the bottom 99 percent, and 0 otherwise. The variable $\mathcal{R}_i = 1 - \mathcal{T}_i - \mathcal{B}_i$ captures the rest of the firms that are not the focus of our longitudinal analysis. Nevertheless, we included them in the regression to estimate the age and time-fixed effects using all the available data. Finally, coefficients α_i and α_t represent firm and year fixed effects, respectively.

Some comments about specification (1) are warranted. First, it is worth emphasizing that the top and bottom classifications are fixed and given by firm size at age 20. Second, the firm fixed effects control for average differences across firms, ensuring that our main coefficients of interest, $\{\beta_a^{\mathcal{T}}\}_{a \in \mathcal{A}}$ and $\{\beta_a^{\mathcal{B}}\}_{a \in \mathcal{A}}$, capture the average life cycle dynamics of top and bottom firms, respectively. Note that these coefficients capture the difference relative to the omitted age group at the firm level. Hence, to interpret the results graphically in the figures below, we scale the results by adding the average value of the variable of interest at age 19-20 for the top and bottom separately. Lastly, to mitigate the impact of outliers in level variables (e.g., output, y) and ratios (e.g., the capital-output ratio, k/y), we estimate (1) using the log of the variable of interest (e.g., $x = \log(y)$). For shares, such as the labor share, wl/y, and the profit share, π/y, we run the regression in levels

[12] In addition to incomplete firm spells, left data censorship is the main reason why we would exclude a firm that we observe at age 20 but do not observe for at least 10 years between age 0 to 20 (e.g., a firm that was age 20 in 1996, we only observe for 5 years in our data from age 16 to 20).

(e.g., $x = wl/y$), excluding values exceeding 2.5 in absolute value.

3.2 Life Cycle Growth

The availability of longitudinal data allows us to move beyond cross-sectional analysis and document novel facts about the origins of top firms. As a first step, we document firms' life cycle growth. Figure 3 presents the output dynamics during the first twenty years of top and bottom firms at age 20, which we estimate using (1).[13] For output, we divide the first age bin into two groups: 0-2 and 3-4. To facilitate interpretation, we normalize the results to the output of the average entrant, defined as firms aged 0-2.

Figure 3 shows remarkably stark differences in the life cycle trajectories of top and bottom firms. Importantly, these differences are present in terms of their *initial size* and life cycle *growth*. On average, firms in the bottom 99 percent at age 20 have a level of output equal to 0.87 times the average entrant at age 0-2, and they grow 1.7 times during the first 20 years. In contrast, firms in the top 1 percent at age 20 are 7.1 times larger than the average entrant at age 0-2, and they grow by a factor of 10.6 times during the first two decades of their life cycle. Hence, top firms at age 20 are not only 8.2 times larger at entry but also grow 6.3 times faster than bottom firms. These stark differences imply that by age 19-20, the average top firm is 75.2 times the size of the average entrant, while the average bottom firm is only 1.5 times the average entrant. The life cycle growth of top and bottom firms is consistent with the view that ex-ante heterogeneity shapes the firm size distribution (Sterk, Sedláček, and Pugsley, 2021).[14]

3.3 Inputs and Profits Over the Life Cycle

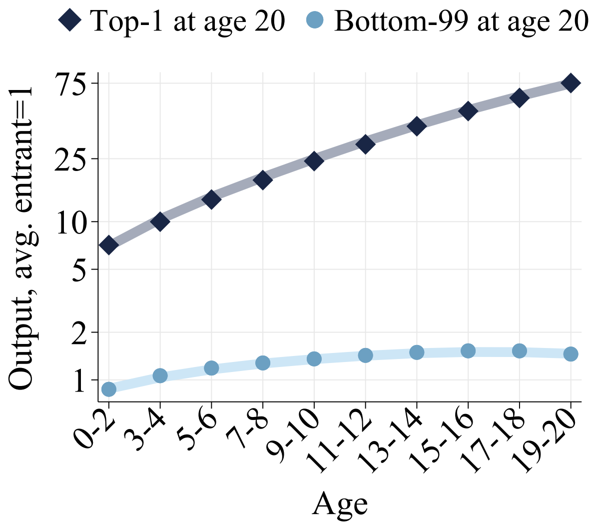
In the previous section, we showed that the firms that reach the top of the size distribution start at a much larger scale and grow significantly faster than the rest during their first 20 years. We now study top firms' input usage (capital and labor) and profit trajectories over the same period, comparing them to bottom firms.

Capital-Output Ratio Panel (a) of Figure 4 presents the trajectory of the capital-output ratio for firms in the top 1 and bottom 99 percent of the size distribution at age 20, obtained from estimating (1). The dynamics of capital relative to output are significantly different between these two groups of firms. Our estimates show that, remarkably, top firms start operating with a considerably high capital stock. The average age 0-4 capital-output ratio of the top 1 percent firms equals 4.8 and is 0.63 log points higher than the

[13] For these results, given the year firm-level fixed effects in (1), we just use log output and not the residualized variable used to classify top and bottom firms.

[14] For example, with mean-reverting shocks and no ex-ante heterogeneity in growth profiles, top firms at age 20 would, on average, have an entry size comparable to that of the typical entrant. This observation stems from the inability of ex-post shocks to generate a high correlation between output at entry and output at age 20 across firms.

Figure 3: Output Over the Life Cycle of Top and Bottom Firms



Notes: Output life cycle trajectories of top 1 and bottom 99 percent firms at age 20 estimated using (1). Output is normalized by the size of the average entrant (age=0-2), and the y-axis is on a log scale. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

ratio at age 19-20. Thus, the capital-out ratio *decreases* over the life cycle of top firms. In contrast, for the bottom 99 percent of firms, the 0-4 capital-output ratio equals 2.4, which is 0.20 log points lower than its age 19-20 value. Hence, the capital-output ratio *increases* as firms age for the bottom 99 of the size distribution. Figure A.6 in the appendix shows that this novel fact on the dynamics of the capital-output ratio for top and bottom firms is robust to alternative measures of capital, such as solely focusing on tangible capital or the sum of tangible and intangible capital.

Labor Share Our second key finding concerns top firms' labor share, defined as labor costs over output. Panel (b) of Figure 4 reports the life cycle dynamics of firms' labor share for the top 1 and bottom 99 percent firms at age 20, estimated using (1). In this case, the initial labor share of top and bottom firms is similar, around 0.70, at ages 0-4. Yet, the evolution of the labor share over firms' life cycles is starkly different for top and bottom firms. For the top 1 percent firms, the labor share *decreases* as firms age. Specifically, the labor share of top firms falls by 11.0 p.p. in the first two decades from 0.70 at age 0-4 to 0.59 at age 19-20. The labor share of the bottom 99 percent of firms is nearly constant over the first 20 years of the firm's life cycle. If something, it slightly increases by 0.1 p.p. from ages 0-4 to 19-20. The main novelty of our findings is that a low labor share is not a fixed characteristic of large firms.[15] Instead, the labor share of firms that end up at the top of the size distribution falls as firms grow. Our findings on

[15]In the cross-section, consistent with Autor et al. (2017), Table 1 shows that the largest firms have a low labor share.

the decline of the labor share for top firms are consistent with the evidence in Mertens and Schoefer (2024), which documents that firms' labor share falls as firms grow over short and medium-term horizons.

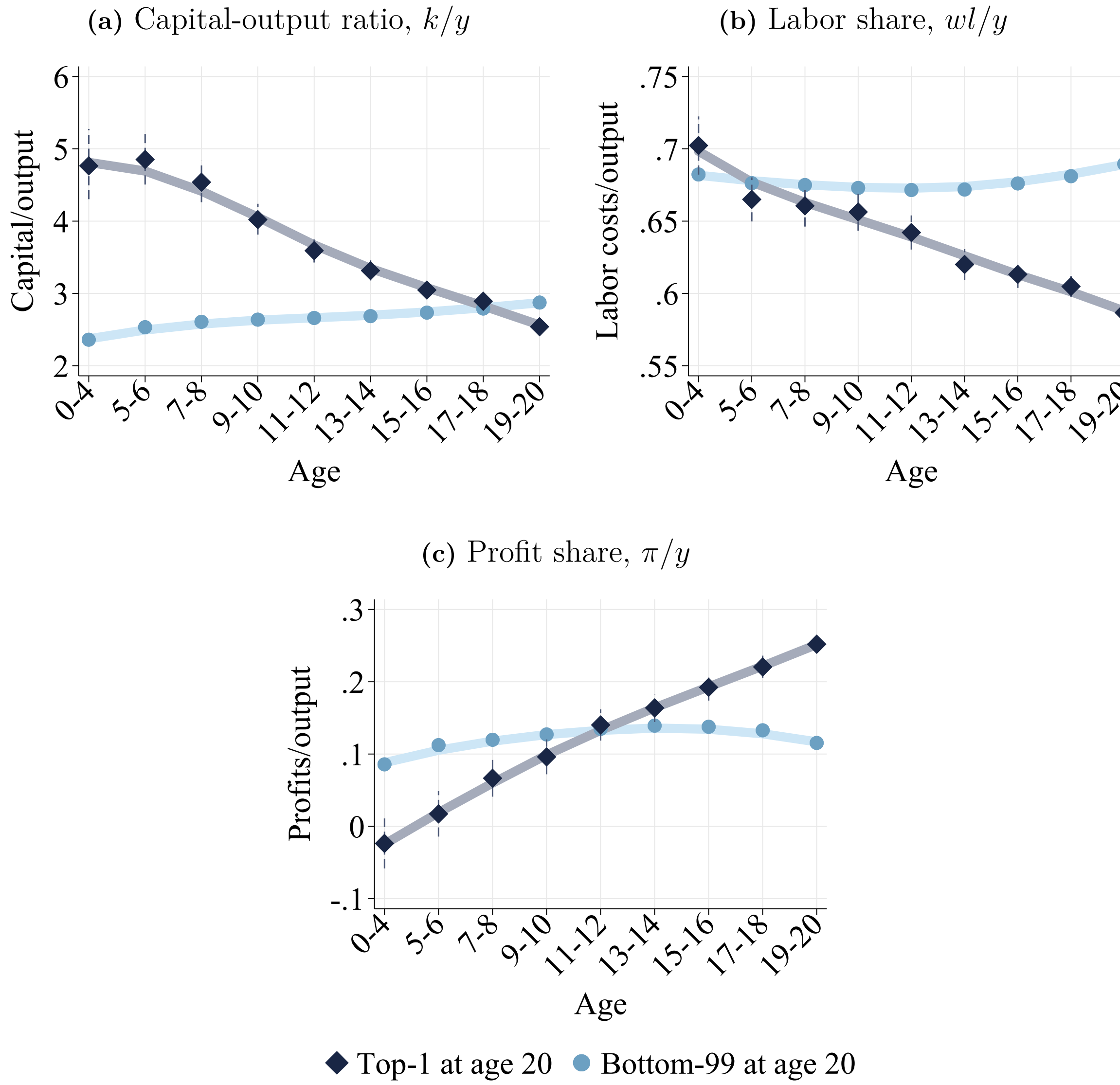
Lastly, it is important to note that the decline in the labor share of top firms is considerably smaller than the decline in the capital-to-output ratio. Indeed, Figure A.14 in the Appendix shows that the capital-to-labor costs ratio significantly declines over the life cycle of top firms. In contrast, bottom firms display a pattern where the increase in the capital-to-output ratio outpaces the almost negligible rise in the labor share over time, leading to an increase in the capital-to-labor costs ratio. As discussed in Section 5, changes in firms' input composition over their life cycle are informative on the factors driving the decline in input usage among top firms over time.

Profit Share After documenting the dynamics of input usage, we now turn our analysis to firms' profits. Panel (c) of Figure 4 presents the results from estimating (1) for the profit share, defined as economic profits over output. The profit share *increases* as firms age for both top and bottom firms. However, the profits of the top 1 percent firms at age 20 are much more *backloaded* towards the second decade of the firms' life cycle. Our estimates indicate that the profit share is not statistically different from zero for the first six years of top firms' life cycles. Yet, the profit share of age 20 top firms significantly increases with firm age and reaches 0.25 by age 19-20. In contrast, the profit share of the bottom 99 percent only increases by 3 p.p., from 0.09 at age 0-4 to 0.12 at age 19-20. Thus, there are remarkable differences in the life cycle dynamics of the profit share by firm size, with top 1 firms' profits having a much steeper life cycle growth than for the bottom 99 percent. Figure A.7 in the appendix shows that this key fact on the life cycle path of the profit share for top and bottom firms is robust to measuring profits by net income, a variable available in the balance sheet data.

In sum, there are substantial differences in the origins of the top 1 and bottom 99 percent of firms at age 20. The findings for the capital-output ratio are consistent with top firms requiring large initial investments, with output subsequently growing faster than capital as firms age. Furthermore, the decline in the labor share over the life cycle of top firms shows that the low labor share in large firms results from significant output growth rather than being a fixed characteristic. Finally, the profit share of top firms has a considerably steeper life cycle growth than bottom firms and is much more backloaded towards the second decade of the firms' life cycle.

In the next section, we extend our analysis to other segments of the firm size distribution, countries, and sectors, and perform various robustness checks. Overall, the main

Figure 4: Inputs and Profit Share Over the Life Cycle of Top and Bottom Firms



Notes: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1). The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

results presented above hold across all these additional exercises.

3.4 Additional Results and Robustness

We now expand on our baseline results across several dimensions. First, we extend our analysis to the average firm and to other segments of the firm size distribution beyond the top 1 percent and bottom 99 percent, as well as to different economies and production sectors. Additionally, we perform various robustness checks, including restricting the sample to privately held firms, employing a balanced panel sample, and reclassifying top firms using alternative firm size variables. Lastly, we present results for materials usage, which exhibit much more modest variation over firms' life cycles.

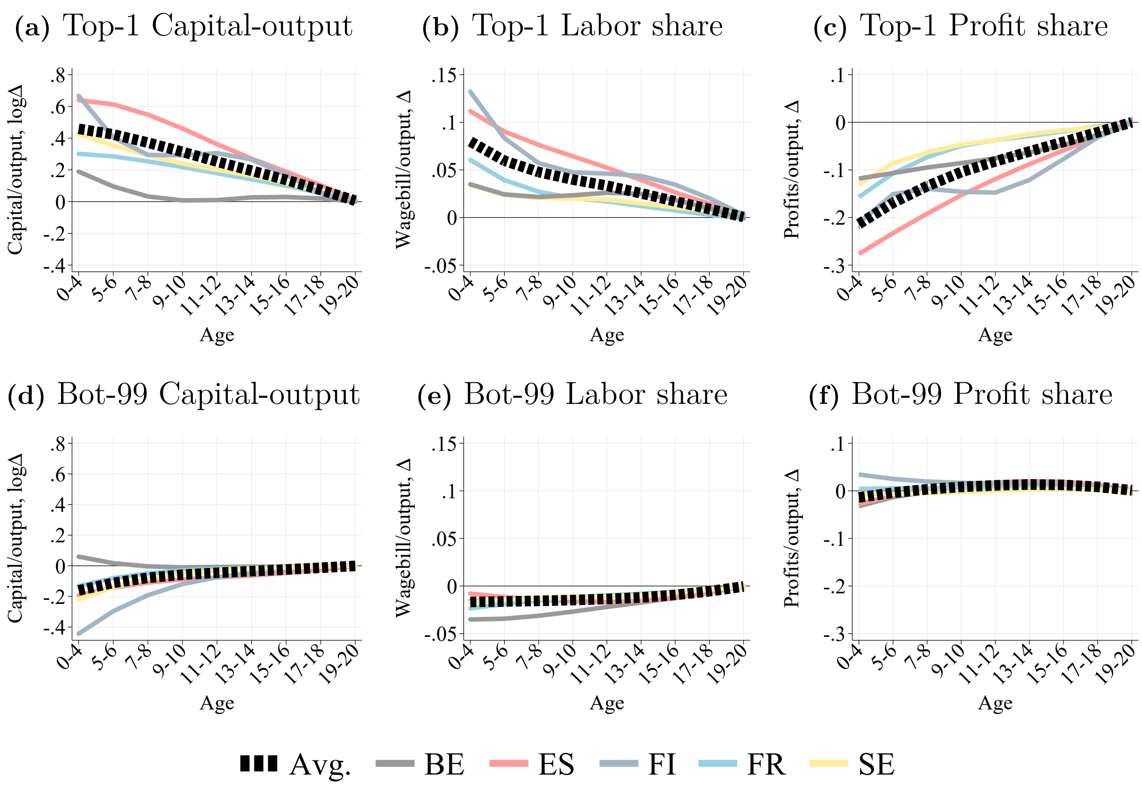
Average Firm First, to motivate the importance of separately studying the life cycle of top and bottom firms, Figure A.8 in the Appendix presents the average trajectory of input usage and profits for firms that survive to age 20. As expected, the life cycle trajectory for the average firm is close to that of the bottom 99 percent, with an increasing capital-output ratio, an even flatter path for the labor share, and a slightly increasing but much less backloaded profit share. As we now show, the average life cycle trajectory of input usage and profits masks substantial heterogeneity among firms at different points of the size distribution, even beyond the top 1 and bottom 99 percent.

Beyond the Top 1 and Bottom 99 Our baseline analysis focuses on the origins of the top 1 and bottom 99 percent of firms at age 20. A natural question is whether our findings extend to other segments of the firm size distribution. To study this, we estimate a version of regression (1) considering six groups of firms defined by their output at age 20: the top 0.01, percentile 99 to 99.9, percentile 95 to 99, percentile 75 to 95, percentile 50 to 75, and the bottom 50 percent. Figure A.9 presents the results for the capital-output ratio, the labor share, and profit share over firms' life cycle, conditional on their size at age 20 according to these six groups. In this case, we present the results for the firm-level changes relative to the age 19-20 value to simplify the exposition. We report log changes for the capital-output ratio and level changes for the labor and profit share.

The main takeaways from Figure A.9 can be summarized as follows. First, this figure shows that our baseline findings on the input and profits life cycle dynamics for top and bottom firms extend to the entire size distribution. Furthermore, the life cycle profiles vary monotonically with firm size, with the top 0.01 and the bottom 50 percent of firms at age 20 exhibiting the starkest differences. Second, it is worth noting that there is heterogeneity even within the top 1 percent, with firms in the top 0.01 exhibiting steeper decreases in the capital-output ratio and labor share, together with much more backloaded profit shares, than firms in the percentile 99 to 99.9. Third, and perhaps more expected, there is significant heterogeneity among bottom firms. Interestingly, the capital-output ratio and labor share increase as firms age for the bottom 50 percent of the size distribution.

Other Countries So far, we have focused on Spain (ES), a country with an ample panel and excellent coverage in Orbis Historical. We now extend our analysis to four additional countries with a sufficiently long panel in Orbis: Belgium (BE), Finland (FI), France (FR), and Sweden (SE). Figure 5 presents the results from estimating (1) for our three main variables of interest separately for each of these five countries, with country-specific classifications for the top 1 and bottom 99 percent firms at age 20. We focus on life cycle changes relative to ages 19–20 to control for average cross-country differences in these variables. The dashed dark line is the average value across countries for the life cycle estimates. Figure 5 shows that our main findings on the origins and life cycle dynamics of top and bottom firms hold in these additional countries. For top 1 percent firms at age 20, we observe a decreasing capital-output ratio, a declining labor share, and a backloaded profit share. In contrast, the bottom 99 percent experience increasing capital-output ratios and labor shares as they age.

Figure 5: Other Countries
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



Notes: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1), separately for each country. Changes are relative to the omitted group (age 19-20). The selected countries are Belgium (BE), Spain (ES), Finland (FI), France (FR), and Sweden (SE). The dashed dark line is the average value across countries for the life cycle estimates. The lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

Sectors Due to data limitations, previous studies have primarily focused on the manufacturing sector (Hsieh and Klenow, 2014; Midrigan and Xu, 2014; Eslava, Haltiwanger, and Urdaneta, 2024). Yet, in developed countries like Spain, the manufacturing sector typically accounts for only 20% to 30% of total output, and its importance has declined over time. An advantage of our data is that it covers all sectors. While we use all firms for our baseline estimates, Figure A.10 in the Appendix presents results for the broad four sectors reported in Table 1: manufacturing, retail and transport, services, and other. This figure shows that our key findings about the first two decades of the top 1 percent firms, of a declining capital-output ratio, decreasing labor share, and more backloaded profits, are present for the top firms in all these sectors.

Private Firms Our analysis includes publicly listed, public companies (C-corporations), and privately held firms. Table 1 reports that listed and public firms constitute 56% of the firms and 77% of the output of the top 1 percent. Hence, a natural question is whether listed and public firms primarily drive our findings. Figure A.11 in the Appendix reports the estimates for the life cycle of top and bottom firms at age 20, differentiating by their legal status: private vs. listed and public firms. This figure shows that our key findings on the life cycle of top firms also hold for the sample of privately held firms.

Balanced Panel For our baseline results, we use a sample of firms observed for at least ten years from ages 0 to 20 to increase the number of observations and have more precise estimates, especially for top firms. As a robustness check, in Figure A.12 in the Appendix, we present the results of estimating (1) restricting to a balanced panel and compare the results with our baseline. The results for the balanced sample, while estimated less precisely, are very similar to our baseline numbers.

Top Firms Under Alternative Measures of Firm Size Throughout our analysis, we defined top and bottom firms using value-added, considering a comprehensive measure of costs. We now consider alternative definitions of top firms using other outcome variables as a robustness check. Figure A.13 in the Appendix presents results from estimating (1) for definitions of top and bottom firms using revenue and total assets at age 20. This figure shows that our main findings on the life cycle dynamics of top and bottom firms are robust to alternative definitions of firm size.

Materials Given the focus of our paper on firms' output, not *revenue*, we measure firm size as revenue minus a comprehensive measure of non-capital and non-labor costs, which includes materials. Contemporaneously developed work has found that materials play a role behind short and medium-run revenue growth (Mertens and Schoefer, 2024) and for the revenue size distribution (Hubmer et al., 2024). To analyze the role of materials usage over the life cycle of top and bottom firms, Figure A.15 in the Appendix presents the results of estimating (1) using materials over revenue as the dependent variable. This figure shows that the materials-to-revenue ratio modestly increases over the life cycle of the top 1 percent of firms at age 20. Yet, the estimates are not statistically different from the age 19-20 value, except for the first age bin. This ratio is roughly constant for the first two decades of firms in the bottom 99 percent at age 20. Thus, we find limited life cycle variation of materials usage at the firm level.

Finance To conclude our life cycle analysis, we document two key facts about finance over the life cycle of top and bottom firms. We focus on firms' net financial leverage, defined as financial debt minus cash over capital, and the rate of equity injections over capital that firms receive from their owners (negative dividends over capital). Figure A.16 in the Appendix presents the results of estimating (1) for leverage and equity injections. Consistent with the findings in Kochen (2023), we document that leverage and equity injections decrease as firms age. Unlike previous work, we find significant differences in the use of debt and equity financing for top and bottom firms. Regarding debt financing, panel (a) of Figure A.16 shows that while leverage is initially similar, around 0.47, the top 1 percent firms at age 20 deleverage *less* over their life cycle, with average leverage of 0.35 by age 19-20 (vs. 0.26 at age 19-20 for the bottom 99 percent). Panel (b) of Figure A.16 shows that top firms' annual equity injections over capital equals 0.049 at age 0-4, twice as large as the number for bottom firms, 0.019. Yet, these differences in equity injections fade out as firms age. In summary, we document that, relative to bottom firms, the top 1 percent of firms at age 20 deleverage less as they age and rely more on equity financing in their early years. The model we describe in Section 4 will be consistent with these facts.

Forward-looking Analysis Alternatively, we perform two forward-looking exercises. First, we pair top firms at age 20 with bottom firms at age 20 that have similar initial characteristics, such as initial size, input usage, profits, and financing. By matching each top firm to a bottom pair and estimating (1) on the paired firms, we study the life cycle patterns of firms that are initially similar to top firms but do not reach the top. Figure A.4 shows the results. We find that their growth, input usage, and profits follow a flat profile. These patterns are consistent with the predictions of our theory for top technologies that are unlucky and do not grow, or for bottom ones that are initially unconstrained.

Second, we study how initial size, input usage, profits, and financing predict whether a firm becomes a top firm twenty years later. Table A.2 presents the results. We find that higher initial output, a higher initial capital-output ratio, and greater initial equity injections make it more likely for a firm to reach the top. To a lesser extent, lower initial profitability and leverage also predict this. These findings hold whether we condition on bottom firms surviving until age 20 or include those that exit before age 20, and they are overall consistent with our baseline exercise, despite it being backward in nature.[16] Appendix A.3 provides further details on both forward-looking empirical exercises.

[16] In other words, if a top firm at age 20 is characterized by certain initial characteristics on average, it does not imply that a firm with these initial characteristics is more likely to be top at age 20, especially considering that some firms may exit before age 20.

3.5 Sources of Ex-Ante Heterogeneity

Having documented stark different trajectories over the life cycle of top and bottom firms, we conclude this section by presenting some suggestive evidence on the sources of firm ex-ante heterogeneity. Using the firm-owner linkages in Orbis, we classify firms according to two key dimensions. First, we study whether the owner is registered in Spain (Domestic owner) or is a foreign individual or company (Foreign owner). Second, we identify whether the owner simultaneously owns multiple firms in the same year. This classification captures both business groups, in the case the ultimate owner is a company, and serial entrepreneurs with individuals as ultimate owners (Brandt et al., 2022; De Vera et al., 2024). Firm-owner linkages are available from 2007 onward. As in the longitudinal analysis, we focus on the first 20 years in the life cycle of top and bottom firms. If a firm is older than 20 years old in the years with available ownership files, we use the first year for which we have ownership data. Not all the firms with available income and balance sheet data have ownership information. However, the data is available for most medium and large firms that have been active from 2007 onward.[17] The following results are at the firm level, which we obtain by averaging over time and owners using ownership equity shares. The results are very similar if we solely focus on firms' majority owners.

Table 2: Firms' Taxonomy by Owners' Characteristics

	By Size at Age 20		
	Top-1	Bot-99	Rest
Domestic owner	0.62	0.80	0.59
Foreign owner	0.33	0.03	0.05
Multiple firms	0.74	0.26	0.23
Multiple firms + Domestic	0.49	0.23	0.19
Multiple firms + Foreign	0.22	0.01	0.03

Notes: Share of firms by owners' characteristics. Domestic (Foreign) owner equals one if the owner is registered in (outside) Spain. Both variables are zero if the owner's country is unknown. Multiple firms is equal to one if the owner owns at least one other firm inside Spain in the same year. Shares are at the firm level, computed after taking each firm's equity-weighted average over different owners and years and then averaging across firms. Results using ultimate owners after three rounds of matching.

Table 2 shows significant differences along these two dimensions of ownership for top and bottom firms. First, regarding domestic and foreign ownership, 33% of the firms in the top 1 percent at age 20 are owned by a foreign entity. In contrast, this number

[17] For Spain, 94% of the age 20 top 1 percent firms have ownership files, while this number is 57% of the age 20 bottom 99 firms. Regarding the rest of the firms, 43% of them have ownership information.

is 3% for the firms in the bottom 99 percent. For reference, we also report results for the rest of the firms (denoted by $\mathcal{R}$ in (1)). Only 5% of the rest of the firms have a foreign owner. Regarding the role of serial entrepreneurs and business groups, the table shows that entities with multiple firms own 74% of the firms in the top 1 percent at age 20.[18] For the bottom 99 percent of firms, 26% of them are owned by owners with multiple firms. Further, entities with multiple firms own 23% of the rest of the firms.[19] The large share of owners with multiple firms in the top 1 percent is partly explained by the over-representation of foreign owners for that group of firms. Yet, domestic entities with multiple firms own 49% of the firms in the top 1 percent at age 20.[20]

In sum, we documented that firms in the top 1 percent at age 20 are almost 11 times more likely to be foreign-owned than firms in the bottom 99. Further, they are 2.8 times more likely to have owners with multiple firms, highlighting the role of business groups and serial entrepreneurs. The firm dynamics model in the next section captures these features of top and bottom firms through ex-ante heterogeneous growth profiles.

4 A Model of Top and Bottom Firms

Motivated by the previous empirical facts, we develop a firm dynamics model with ex-ante heterogeneity, non-homothetic production technologies, and forward-looking debt and equity financing to explain our findings. We use the model to explain our empirical findings and study the macroeconomic implications of the origins of top firms.

4.1 Setup

We study a discrete-time, infinite-horizon economy inhabited by heterogeneous firms. Technologies are scarce, so each period, a mass of potential entrants, $\mathcal{E}$, decides whether to enter the market and begin operations. Firms are born with a technology type $h \in \mathcal{H}$, where the measure of types $\mathcal{E}_h$ satisfies $\sum_h \mathcal{E}_h = \mathcal{E}$.[21] The technology types can vary in their productivity process, returns to scale, input mix, and input non-homotheticities. The mass of potential entrants and their distribution across types are exogenous, characterizing the technological frontier of the economy.

[18] The prevalence of owners with multiple firms in the top 1 percent is consistent with Blum et al. (2020), which documents that existing businesses own 70% percent of new exporters in Chile.

[19] Our findings for Spain are consistent with the literature on serial entrepreneurs (SE). Brandt et al. (2022) documents that around 30% of the firms in China are owned by SE, while De Vera et al. (2024) find that this number is 13.5% in Portugal.

[20] Figure A.17 in the Appendix shows the life cycle patterns of top firms conditional on these two owners' characteristics. While our main results are robust to different subsets of ownership, foreign-owned and firms with multi-firm owners exhibit starker reductions in the capital-output ratio as firms age, consistent with these firms having large initial investments early in their life cycle.

[21] The technological scarcity can also be understood as the presence of implicit adoption costs required for technologies to begin operating.

Potential entrants are drawn from a time-invariant distribution G over firms' idiosyncratic states—technology type, assets, and productivity. If potential entrants decide to enter, they produce a unique final good using capital and labor. Aside from production, they can also save in a risk-free asset. The risk-free interest rate r is exogenous, while wages w are determined in equilibrium. To close the labor market, there is an aggregate labor supply function $L^s(w)$, which in equilibrium equals firms' aggregate labor demand L^d, determining the wage w. Prices remain constant, as we focus on the steady-state equilibrium, where t indexes firm age.

We now provide a detailed description of firms' objective, production technology, financial frictions, and their entry and exit.

Firms' Objective Firm i at age t maximizes the net present value of its dividends, d_{it}:

$$V_{it} = \mathbb{E}_t \left[\sum_{s \geq t} \beta^{s-t} d_{i,s} \right]$$

for $t \geq 0$, where $\beta \in (0,1)$ is the firm's subjective discount factor and satisfies $\beta(1+r) \leq 1$.

Productivity Process The evolution of firms' productivity is technology-specific, with the productivity process of firm i with technology h at age $t \geq 1$ given by

$$p^h \left(\mathbf{s}_{it}^h \right) = \exp \left(u_{it}^h + z_{it} \right) \tag{2}$$

where $\mathbf{s}_{it}^h = (u_{it}^h, z_{it})$, u_{it}^h is the *ex-ante* component, and z_{it} is the *ex-post* component.

Similar to Sterk, Sedláček, and Pugsley (2021) and Hopenhayn, Neira, and Singhania (2022), we introduce ex-ante heterogeneity in the TFP process by assuming that, when born, firms draw an initial ex-ante component $u_{i0}^h \sim \mathcal{N} \left(\underline{\mu}_u^h, \underline{\sigma}_u^2 \right)$, which evolves *deterministically* as

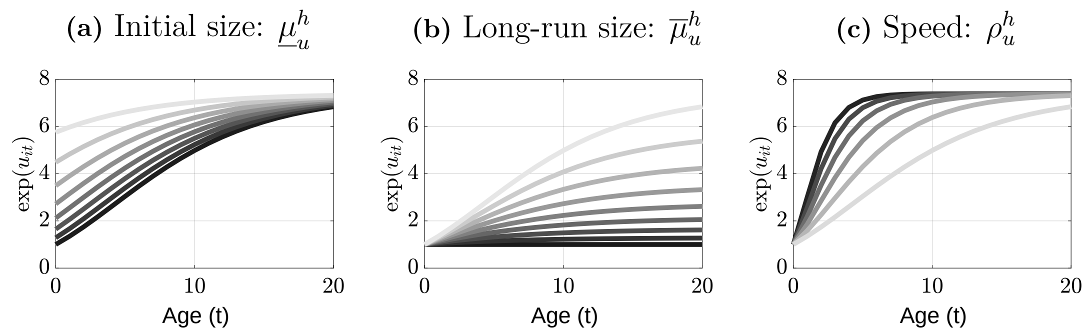
$$u_{it+1}^h = \rho_u^h u_{it}^h + \left(1 - \rho_u^h \right) \overline{\mu}_u^h$$

where the parameters $\left(\rho_u^h, \underline{\mu}_u^h, \overline{\mu}_u^h \right)$ jointly determine the level and growth profile of h. Specifically, $\underline{\mu}_u^h$ defines the initial size across types, $\overline{\mu}_u^h$ specifies the size in the long run, and ρ_u^h determines the speed at which the TFP ex-ante component converges to its long-run value (see Figure 6). Lastly, $\underline{\sigma}_u^2$ represents the degree of ex-ante heterogeneity within technologies.

Given an initial value z_{i0}, the ex-post component of TFP evolves *stochastically* according to the law of motion

$$z_{it+1} = \rho_z z_{it} + \varepsilon_{it+1},$$

Figure 6: Ex-ante TFP Component



Notes: The figure shows different paths of the deterministic ex-ante TFP component $\exp(u_{it})$ for different values of $\underline{\mu}_u^h$ (panel a), different values of $\overline{\mu}_u^h$ (panel b), and different values of $\overline{\mu}_u^h$ (panel c). Lighter lines indicate higher values. Numerical example.

where ex-post shocks are $\varepsilon_{it+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Hence, notice that a firm can be in the top 1 percent of the size distribution firm if it is lucky (i.e., experiences a sequence of large ex-post shocks) and/or has a high-growth and level profile.

Production Function Firms combine capital k_{it} and labor l_{it} to produce output y_{it} using technology

$$y_{it} = p\left(\mathbf{s}_{it}^h\right) f^h\left(k_{it}, l_{it}\right).$$

The function $f^h(k_{it}, l_{it})$ exhibits decreasing returns to scale (DRS) in capital and labor, aggregating the inputs using a constant elasticity of substitution (CES) aggregator. Additionally, there are non-homotheticities in capital and labor (Geary, 1950; Stone, 1954).[22] The function $f^h(k_{it}, l_{it})$ is parameterized as

$$f^h\left(k_{it}, l_{it}\right) = \left[\alpha_h^{\frac{1}{\sigma}}\left(k_{it} - \kappa_k^h\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\alpha_h\right)^{\frac{1}{\sigma}}\left(l_{it} - \kappa_l^h\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}\theta^h} \tag{3}$$

where σ is the elasticity of substitution between capital and labor (with $\sigma < 1$ indicating complements and $\sigma > 1$ indicating substitutes), α_h is the capital share, and $\theta^h \in (0,1)$ represents the degree of decreasing returns to scale (DRS), which may depend on the technology type.[23] Central to our analysis are the technology-specific non-homotheticities in capital and labor, parameterized by κ_k^h and κ_l^h, respectively. In Appendix B, we discuss how input non-homotheticities can account, for example, for production input indivisibilities and complementarities with TFP.

[22]Other types of parametric non-homothetic production functions have been used by Lau and Tamura, 1972 (Leontief) and Hanoch (1975), Sato (1977), Lashkari, Bauer, and Boussard (2024), and Eckert, Ganapati, and Walsh (2024) (non-homothetic CES).

[23]The CES production function nests the Cobb-Douglas case, given by $f^h(k_{it}, l_{it}) = \left[\left(k_{it} - \kappa_k^h\right)^{\alpha_h}\left(l_{it} - \kappa_l^h\right)^{1-\alpha_h}\right]^{\theta^h}$, which can be obtained from (3) when $\sigma \to 1$.

Production Costs Firms hire labor at wage w and rent capital at rate R, such that the firms variables costs are $wl_{it} + Rk_{it}$. Capital depreciates at an exogenous rate $\delta \in [0,1]$, so in equilibrium, the rental rate of capital is $R = \delta + r$, while wages are determined by firms' labor demand and the aggregate labor supply function. As a result, at period t, the firm i profits are $\pi_{it} = y_{it} - Rk_{it} - wl_{it}$. Although it is not explicit, it is important to note that non-homotheticities are equivalent to input-specific fixed costs. Using $\tilde{k}_{it} = k_{it} - \kappa_k^h$ and $\tilde{l}_{it} = l_{it} - \kappa_l^h$, such that $\tilde{f}^h(\tilde{k}_{it}, \tilde{l}_{it})$ is a standard homothetic CES aggregator, we can rewrite profits as $\pi_{it} = y_{it} - (R\tilde{k}_{it} + w\tilde{l}_{it}) - (R\kappa_k^h + w\kappa_l^h)$, where $R\kappa_k^h + w\kappa_l^h$ represents the input-specific fixed costs.

Financial Assets and Frictions Over time, firms can accumulate a risk-free asset $a_{it} \geq 0$, which earns a return r. This asset can be used as collateral and liquidated in any given period. There are two sources of financial frictions. First, it is costly to issue equity (Hennessy and Whited, 2007). Dividends are given by $d_{it} = \tilde{d}_{it} - \Lambda\left(\tilde{d}_{it}\right)$, where $\tilde{d}_{it} = \pi_{it} + a_{it}(1+r) - a_{it+1}$ are the dividends before accounting for equity issuance costs and $\Lambda\left(\tilde{d}_{it}\right)$ the equity issuance costs. The issuance cost function is given by

$$\Lambda\left(\tilde{d}_{it}\right) = \begin{cases} \chi_0 + \chi_1 \left|\tilde{d}_{it}\right| & \text{if } \tilde{d}_{it} < 0 \\ 0, & \text{otherwise,} \end{cases}$$

where $\chi_0, \chi_1 \geq 0$ are positive parameters. Second, firms can rent a limited amount of capital $k_{it} \leq \bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$. We parameterize the maximal amount of capital that a firm can rent, $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$, as

$$\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right) = \underbrace{\lambda_a a_{it}}_{\text{backward-looking}} + \underbrace{\lambda_v \max\left\{0, V^{h*}\left(\mathbf{s}_{it}^h\right)\right\}}_{\text{forward-looking}}, \tag{4}$$

where $\lambda_a \geq 1$ captures how much capital can be rented using assets as collateral (Kiyotaki and Moore, 1997), which is backward looking, and $\lambda_v \geq 0$ is how much additional capital can be rented against the value of the firm, which is forward looking (Albuquerque and Hopenhayn, 2004; Brooks and Dovis, 2020). For simplicity, we assume that the forward-looking component of the collateral constraint depends on the firm's unconstrained value, $V^{h*}(\mathbf{s}_{it}^h)$, which we define later.[24] This value reflects both current and future profits. The equity issuance and the forward-looking borrowing allows firms to fund themselves against current assets and future expected profits, which can vary significantly depending

[24]For tractability and to avoid potential multiplicity issues, we assume the constraint depends on the unconstrained value rather than the baseline value. Since the constraint also depends on assets, it reflects the firm's current asset position, as the regular firm value, which incorporates both assets and future profits.

on different growth profiles. Importantly, the set of parameters (χ_0,χ_1,λ_a,λ_v) governing the degree of financial development are common across technology types.

Entry Firms are born with a technology type h, which has a measure $\mathcal{E}_h$, and draw initial productivity u_{i0}^h and assets a_{i0} from a distribution $G^h\left(a_{i0}, u_{i0}^h\right)$.[25] Following Kochen (2023), u_{i0} is drawn from $u_{i0}^h \sim \mathcal{N}\left(\underline{\mu}_u^h, \underline{\sigma}_u^2\right)$. Once u_{i0} is determined, the firm's initial asset position is given by $a_{i0} = \iota k_0^{h*}\left(u_{i0}^h, 0\right)$, where $k_0^{h*}\left(u_{i0}^h, 0\right)$ represents the initial unconstrained level of capital, and $\iota \in [0,1]$ indicates how close the firm is from initially having enough assets to be unconstrained, which is drawn from $\iota \sim \text{Beta}\left(\bar{\iota}, 1\right)$. Thus, conditional on h, the parameters $\{\underline{\mu}_u^h, \sigma_u^2, \bar{\iota}\}$ characterize the distribution $G^h\left(a_{i0}, u_{i0}^h\right)$. After being born, the firm decides whether to enter. If it enters, it starts operating in the next period; otherwise, it exits with value $V_e\left(a_{i0}\right) = (1+r)a_{i0}$.

Exit Firms can exit for several reasons. First, as in standard models of firm dynamics with financing frictions, given a type h, the firm is forced to exit with an exogenous probability $\eta^h \in [0,1]$.[26] Second, the firm may decide to exit if the value of continuing V^h (defined later) is smaller than the value of exiting $V_e\left(a_{it}\right)$. Finally, the firm may exit if it not able to satisfy the capital non-homotheticity, $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right) < \kappa_k$.[27] In all cases, the exit value is equivalent to the liquidation value of the assets, $V_e\left(a_{it}\right) = (1+r)a_{it}$.

4.2 Firm Problem

The firm solves a static profit maximization problem and a dynamic savings decision, which can be written recursively.

Profit Maximization Given the idiosyncratic states $(a_{it}, \mathbf{s}_{it}^h)$, a firm i with type h technology has per-period optimal profits given by

$$\begin{aligned} \pi^h\left(a_{it}, \mathbf{s}_{it}^h\right) = & \max_{k_{it}, l_{it}} \quad p^h\left(\mathbf{s}_{it}^h\right) f^h\left(k_{it}, l_{it}\right) - Rk_{it} - wl_{it} \\ & \text{s.t.} \quad k_{it} \leq \bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right), \end{aligned} \tag{5}$$

such that he capital $k^h\left(a_{it}, \mathbf{s}_{it}^h\right)$ and labor $l^h\left(a_{it}, \mathbf{s}_{it}^h\right)$ policy functions solve this problem. As a benchmark and to compute the firm's collateral constraint, it is helpful to define the unconstrained solution. The unconstrained profits $\pi^{h*}\left(\mathbf{s}_{it}^h\right)$ are defined as the solution to problem (5) when capital is not subject to the collateral constraint. The policy functions $k^{h*}\left(\mathbf{s}_{it}^h\right)$ and $l^{h*}\left(\mathbf{s}_{it}^h\right)$ denote the optimal unconstrained choices of capital and

[25] Note that G represents the initial distribution across and within types, while G^h is the distribution conditional on type h.

[26] This assumption is common in models of firm dynamics with financial frictions, as becoming unconstrained is an absorbing state when $\beta(1+r) = 1$.

[27] In other words, we do not allow the firm to remain dormant until it satisfies this condition or chooses to exit.

labor, respectively. To find collateral constraint $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$, we compute iteratively the unconstrained value $V^{h*}\left(\mathbf{s}_{it}^h\right) = \max \pi^{h*}\left(\mathbf{s}_{it}^h\right) + \left(1-\eta^h\right)\beta\mathbb{E}\left[\max\left\{V^{h*}\left(\mathbf{s}_{it+1}^h\right), 0\right\}\right]$ using the unconstrained profits. Appendix B provides more detail of the constrained and unconstrained solutions.

Incumbent Recursive Problem Let $\tilde{V}^h$ be the value of the incumbent firm i of technology type h with states $\left(a_{it}, u_{it}^h, z_{it}\right)$ at the beginning of period t such that

$$\tilde{V}^h\left(a_{it}, u_{it}^h, z_{it}\right) = \max\{\underbrace{V^h\left(a_{it}, u_{it}^h, z_{it}\right)}_{\text{continue}}, \underbrace{V_e\left(a_{it}\right)}_{\text{close}}\}, \tag{6}$$

where $V_e(a_{it}) = a_{it}(1+r)$ and V^h denotes the value of continuing operations instead of closing the firm. It follows that the value of continuing V^h is given by

$$V^h\left(a_{it}, u_{it}^h, z_{it}\right) = \max_{a_{it+1},\ \tilde{d}_{it}} \tilde{d}_{it} - \Lambda(\tilde{d}_{it}) + \beta\mathbb{E}\left[\left(1-\eta^h\right)\tilde{V}^h\left(a_{it+1}, u_{it+1}^h, z_{it+1}\right) + \eta^h V_e\left(a_{it+1}\right)\right] \tag{7}$$

where

$$\begin{aligned} \tilde{d}_{it} &= \pi^h\left(a_{it}, u_{it}^h, z_{it}\right) + a_{it}\left(1+r\right) - a_{it+1} \\ u_{it+1}^h &= \rho_u^h u_{it}^h + \left(1-\rho_u^h\right)\overline{\mu}_h \\ z_{it+1} &= \rho_z z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim N\left(0, \sigma_\varepsilon^2\right). \end{aligned}$$

If there is a feasible solution to the incumbent problem (i.e., $\bar{k}^h(a_{it}, \mathbf{s}_{it}^h) \geq \kappa_k$ is satisfied), the policy function $a^h\left(a_{it}, u_{it}^h, z_{it}\right)$ solves this problem. Otherwise, the firm exits with $V_e(a_{it})$.

Entry Problem A firm enters if the expected value of entering and operating exceeds the outside option of retaining the initially endowed assets. Thus, the value of the potential entrant i with technology h and states (a_{i0}, u_{i0}^h) is

$$V_0^h\left(a_{i0}, u_{i0}^h\right) = \max\{\underbrace{\mathbb{E}\left[\tilde{V}^h\left(a_{i0},\ \rho_u^h u_{i0}^h + \left(1-\rho_u^h\right)\overline{\mu}_h, \varepsilon_z\right)\right]}_{\text{enter}}, \underbrace{a_{i0}(1+r)}_{\text{not enter}}\}. \tag{8}$$

where the value of not entering is keeping the firm wealth plus the risk free return. Importantly, unlike the free-entry condition, technologies are scarce. The availability of different technologies, represented formally by the measures $\{\mathcal{E}_h\}_{h\in\mathcal{H}}$, can be interpreted as the technological frontier of the economy. Changes in the technological composition—formally, the distribution of these measures—can have significant aggregate implications, which we study in our quantitative applications.

4.3 Competitive Equilibrium

A *stationary competitive equilibrium* in this economy consists of: (i) wage w and interest rates $\{r, R\}$; (ii) incumbent firms' policy functions $\left\{a^h\left(a_{it}, \mathbf{s}_{it}^h\right), k^{h*}\left(\mathbf{s}_{it}^h\right), l^{h*}\left(\mathbf{s}_{it}^h\right)\right\}_{h\in\mathcal{H}}$; (iii) potential entrants' entry and incumbents' exit decisions; (iv) initial distribution of idiosyncratic states G, types measures $\{\mathcal{E}_h\}_{h\in\mathcal{H}}$ and distribution of incumbent firms that produce $\Omega^h\left(a_{it}, \mathbf{s}_{it}^h\right)$; and (v) aggregate labor supply function $L^s(w)$ such that:

- Given prices, the policy functions solve the incumbent firms' profit maximization (5) and dynamic problem (7).
- Entry choices are consistent with problem (8), and exit decisions are consistent with the exit decision (6) and the capital non-homotheticity feasibility.
- The labor market clears, i.e., $L_s(w) = \sum_{h\in\mathcal{H}} \int l^h\left(a_{it}, \mathbf{s}_{it}^h\right) \mathrm{d}\Omega^h$.
- The rental rate of capital is determined by the risk-free rate and the capital depreciation rate such that $R = r + \delta$.
- For each $h \in \mathcal{H}$, the distribution $\Omega^h(a_{it}, \mathbf{s}_{it}^h)$ is implicitly defined by the equilibrium mapping $\mathbf{\Omega}^h = \mathcal{T}\left[\mathbf{\Omega}^h\right]$, which is determined by the exit and entry decisions, policy functions, and the initial distribution of firms.

5 Firms Input Usage in Data and Theory

In this section, we contrast the model's implications for input usage over the life cycle with our empirical findings for top and bottom firms. Additionally, we analyze alternative hypotheses related to changes in product and labor market power over the life cycle, as well as input-biased growth, which are abstracted from in our baseline model.

5.1 Input Usage and Profits in the Baseline Model

Our stylized theory provides two potential sources through which the capital-output, labor share, and input mix can vary across the life-cycle. Let μ_i be the lagrange multiplier on the capital collateral constraint in problem (5), then the capital-output ratio in the model is

$$\frac{k_{it}}{y_{it}} = \theta^h \alpha_h \underbrace{\left(\frac{1}{R+\mu_{it}}\right)}_{\text{capital costs}} \underbrace{\left[\alpha_h + (1-\alpha_h)\left(\frac{w}{R+\mu_{it}}\right)^{1-\sigma}\right]^{-1}}_{\text{input complementarities}} \underbrace{\frac{1}{\left(1-\kappa_k^h/k_{it}\right)}}_{\text{K non-homotheticity}}. \tag{9}$$

The first term on the right-hand side of (9) depends on the returns to scale θ^h and capital elasticity α_h, which are fixed over time.

The second term relates to the cost of capital. The higher the cost, the lower the capital-output ratio. The cost of capital consists of the rental rate, which is fixed over time, and the shadow cost of capital, which reflects how binding the capital collateral constraint is. The shadow cost of capital can vary over a firm's life cycle. For example, as firms age and become less constrained, μ_{it} decreases, leading to an increase in the capital-output ratio. Given a decline in the shadow cost of capital, the magnitude of this increase depends on the elasticity of substitution σ, with a higher σ implying a faster increase in k/y. An increase in the capital-output ratio as firms age aligns with the empirical observations for the bottom firms but not for the top firms, as we document in the empirical section.

An additional source of variation in the capital-output ratio comes from input non-homotheticities. For firms that produce, we know that $\kappa_k^h > \overline{k}^h \geq k_{it}$, which implies that $\kappa_k^h/k_{it} \in [0,1)$ and that $\frac{1}{\left(1-\kappa_k^h/k_{it}\right)} \geq 1$. Importantly, as k_{it} increases this term will decrease. For example, if firms initially operate with levels of capital close to the necessity level κ_k^h, then the k/y ratio will be large at the start, but as they grow, it will gradually decline. This pattern aligns with the life-cycle dynamics of k/y for top firms, suggesting, through the lens of the theory, that non-homotheticities may play a significant role for firms that reach the top of the size distribution, while potentially being much less important for bottom firms.

Now we turn to the labor share. In the model, the firm-level labor share is

$$\frac{wl_{it}}{y_{it}} = \theta^h \left(1-\alpha_h\right) \underbrace{\left[\left(1-\alpha_h\right) + \alpha_h \left(\frac{R+\mu_{it}}{w}\right)^{1-\sigma}\right]^{-1}}_{\text{input complementarities}} \underbrace{\frac{1}{\left(1-\kappa_l^h/l_{it}\right)}}_{\text{L non-homotheticities}} . \tag{10}$$

Analogous to the k/y ratio, there is a fixed component that depends on the shape of the production function, $(1-\alpha_h)$ and θ^h, which remains constant over time. Although labor is not directly distorted by financial frictions, it is indirectly affected by them if the input aggregator is not Cobb-Douglas ($\sigma \neq 1$). Moreover, the labor share can also vary if there are labor input non-homotheticities ($\kappa_l^h > 0$).

Next, we discuss how financial frictions can distort the labor share. Assuming that financial frictions become less relevant as firms grow and μ_{it} decreases, the change in the labor share will depend crucially on the parameter σ, which determines the degree of complementarity and substitution between capital and labor. A sensible assumption is that as firms age and grow, they become less financially constrained, reducing μ_{it}. In this case, if $\sigma > 1$, the labor share would decrease as labor is substituted away in favor of capital, which becomes relatively cheaper. In contrast, if $\sigma < 1$, the labor share would increase, as capital and labor are complements. A third possibility is that $\sigma \approx 1$, in which case the labor share remains roughly unchanged. This last possibility is consistent with the dynamics of the labor share of bottom firms, which experience an increase in k/y over time while maintaining a stable labor share. On the other hand, for top firms, labor and capital must be substitutes to match the decline in the labor share; however, this would counterfactually imply an increase in k/y.

There is an additional term affecting the labor share, which depends on the necessary level of labor required for production, κ_l^h. This term implies that if firms start with employment levels close to κ_l^h they will have initially a large labor share, which over time will decline as firms grow. This pattern is consistent with the labor share trajectories we observe for top firms, but not for bottom firms. Similar to the capital-output ratio, the model suggests that a stylized explanation for the labor share decline among top firms is the presence of input non-homotheticities in high-growth technology types. In addition, we observe that k/wl declines sharply for top firms, suggesting that capital non-homotheticities are larger than labor non-homotheticities.[28]

Finally, the profit share is

$$\frac{\pi_{it}}{y_{it}} = 1 - \underbrace{\left(\frac{wl_{it}}{y_{it}} + R\frac{k_{it}}{y_{it}}\right)}_{\text{inputs usage}} = 1 - \underbrace{\left(\frac{w\tilde{l}_{it}}{y_{it}} + R\frac{\tilde{l}_{it}}{y_{it}}\right)}_{\text{inputs usage net of non-h}} - \underbrace{\left(\frac{w\kappa_l^h}{y_{it}} + R\frac{\kappa_k^h}{y_{it}}\right)}_{\text{input-specific fixed costs}} . \tag{11}$$

This share will be shaped by the input usage, i.e., the sum of the labor share and capital share. For example, if input usage is decreasing over the life-cycle, the profits path over the life cycle is going to be backloaded, as we observe for top firms. Importantly, non-homotheticities will show up as an input-specific fixed cost that will have a lower importance as firms grow. The input usage costs not related to non-homotheticities may vary across the life-cycle if distortions, such as financial frictions, change over the life cycle.

Overall, decreasing input usage and backloaded profits over the life cycle due to high growth and non-homotheticities, especially in capital, are consistent with the origins and trajectories of top firms.[29] On the other hand, increasing capital usage, combined with a flat labor share and a mildly decreasing profit share due to financial frictions limiting capital usage in the initial years, along with capital-labor substitution of approximately $\sigma \approx 1$, is consistent with the life cycle of bottom firms.

[28]The capital-to-labor costs ratio is $\frac{k_{it}}{wl_{it}} = \frac{\alpha_h}{1-\alpha_h}\left(R+\mu_{it}\right)^{-\sigma} w^{\sigma-1}\frac{\left(1-\kappa_l^h/l_{it}\right)}{\left(1-\kappa_k^h/k_{it}\right)}$.

[29]It is important to note that the non-homothetic technology explanation for the life-cycle input usage of top firms does not depend on the elasticity between capital and labor, a value that remains highly contested (see, for example, Karabarbounis and Neiman (2013) and Oberfield and Raval (2021)).

5.2 Alternative Theories of Top Firms' Declining Input Usage

We now discuss alternative explanations for the life-cycle pattern of input usage among top firms. In Appendix B.2, we extend the static firm problem by allowing firms' output prices and wages to be elastic to their output and labor choices. Changes in these elasticities can capture, for example, changes in output and labor market power as firms age.[30] Furthermore, we allow for TFP growth to be labor and capital augmented. As we focus on explanations beyond non-homotheticities for the top firms' decline in input usage, we assume that technology is homothetic and there are no financial frictions for these cases. Appendix B.2 analytically derives the input-to-output ratios for these extensions of the model. Table 3 summarizes the main results regarding input usage over firms' life cycle in the data, the baseline model, and under these alternative theories.

First, increasing markups as firms grow can explain the decline in k/y and wl/y. Nevertheless, variable markups imply a proportional change in both input-to-output ratios while maintaining the input mix unchanged. This prediction is at odds with the data where k/wl significantly declines. Second, an increase in the shadow cost of labor as firms grow, as in Mertens and Schoefer (2024), can explain the decline in the labor share. The capital-output ratio will also fall if capital and labor are complements ($\sigma < 1$). However, such complementarities would imply a smaller decrease in k/y relative to wl/y (see (24) in the Appendix), which is once again inconsistent with top firms' life-cycle patterns. More generally, any combination of changes in labor and product market power over the life cycle of firms would be unable to account for the observed decline in k/wl.

Lastly, regarding input-augmenting TFP growth, equations (22) and (23) in the Appendix show that the capital-output and labor share depend on the capital-to-labor augmented TFP ratio only if $\sigma \neq 1$. Regardless of the value of σ, capital-biased growth will cause the labor share and the capital-output ratio to move in opposite directions. Therefore, changes in capital- and labor-augmented TFP cannot account for the simultaneous decline in the capital-output ratio and the labor share of top firms.

In sum, as Table 3 shows, it is challenging for these alternative theories to explain the decline in top firms' input usage over their life cycle. We conclude that non-homotheticities in inputs combined with high growth offer a parsimonious explanation for the joint declines in k/y and wl/y among top firms, without relying on a convoluted combination of factors or taking a stance on whether capital and labor are complements or substitutes.

[30] Changing elasticities can also reflect variations in input adjustment costs over time.

Table 3: Input Usage Over Firms' Life Cycle in Data, Model, and Alternative Theories

		Life Cycle Δ		
	σ	k/y	wl/y	k/wl
Data				
Top 1 percent firms		$(-)$	$(-)$	$(-)$
Bottom 99 percent firms		$(+)$	$(\approx)$	$(+)$
Baseline Model				
High growth + non-homotheticity*	any value	$(-)$	$(-)$	$(-)$
K shadow costs decrease	$\sigma \approx 1$	$(+)$	$(\approx)$	$(+)$
Alternative Theories for Top Firms				
Markups increase	any value	$(-)$	$(-)$	$(=)$
L shadow costs increase	$\sigma < 1$	$(-)$	$(-)$	$(+)$
K-biased growth	$[\sigma < 1, \sigma > 1]$	$[(-),(+)]$	$[(+),(-)]$	$[(-),(+)]$

Notes: $(+)$ implies that the variable increases over firms' life cycle, $(-)$ decreases, and $(=)$ denotes no change. *Data* refers to the evidence documented in Section 3, *Model* to the baseline model implications, and *Alternative Theories for Top Firms* to the alternative hypotheses discussed in the text for the top firms input composition. *For k/wl to decline, κ_k must be large relative to κ_l.

6 Parameterization

This section describes our calibration strategy. First, we assign some parameters to standard values in the literature. Second, we calibrate 27 parameters, some common and some technology h-specific, so the model is consistent with 92 moments from the data. We calibrate the model to Spain, targeting standard firm dynamics moments and the key novel facts of top and bottom firm dynamics documented in Section 3.

6.1 Assigned Parameters

We set the interest rate $r = 0.02$, and firms' subjective discount factor to $\beta = 1/(1.04)$. For the capital depreciation rate we use a value of $\delta = 0.05$. For simplicity, we assume $H = 2$. Hence, there are two types of technologies: h=1 denotes the high-growth technologies, and h=2 the bottom, modest-growth technologies. We set the exogenous probability of firm exit by technology type to $(\eta^1, \eta^2) = (0.04, 0.06)$ such that, beyond endogenous exit, the technological composition improves as firms age.

6.2 Calibrated Parameters and Model Fit

We calibrate the remaining parameters so that the model replicates key cross-sectional and longitudinal moments and those capturing top and bottom firm dynamics. Table 4 presents the 92 moments used in the calibration. Table 5 reports the 26 calibrated parameter values. We first describe the targeted moments, second explain our calibration strategy, and third describe the model fit.

Targeted Moments Panel (a) of Table 4 includes the more standard, cross-sectional, and longitudinal firm dynamics moments we target in our calibration exercise. These moments include cross-sectional exit rates by firm age and size, the total output share of the top 1 and top 10 percent of firms, the average and standard deviation of output growth, and the autocorrelation of log output at different horizons. Panel (b) reports the key novel moments on top and bottom firm dynamics which we directly target. We compute these moments in the model-simulated data using the same strategy as in the empirical section.[31] The first set of moments includes the initial level and growth of output, the capital-output ratio, labor share, and profit share over the life cycle of bottom and top firms at age 20, which we documented in Section 3. The second set of moments relates to firms' debt and equity financing. We target the average leverage of top and bottom firms and their equity injections relative to capital at ages 0-4 and 19-20.

Calibrated Parameters Table 5 reports the calibrated parameters. We classify the parameters into four groups: productivity, technology, entrants, and finance. We make some initial assumptions to reduce the dimension of the parameter space. First, without loss of generality, we assume that the bottom technologies' initial ex-ante TFP equals $\underline{\mu}_u^2 = 0$. Second, as we analytically show in Section 5, our model requires an elasticity of substitution between capital and labor close to unity, so we set $\sigma = 1$ for both types of technologies. Third, given the life cycle trajectory of the capital-output ratio and the labor share for the bottom 99, we set the non-homothetic input costs of h=2 technologies to zero: $(\kappa_k^2, \kappa_l^2) = \mathbf{0}$. Lastly, without loss of generality, we fix the mass of potential entrants every period to $\mathcal{E} = 1$, and hence we have that $\mathcal{E}_2 = 1 - \mathcal{E}_1$.

After the previous assumptions, we are left with 21 parameters. We find the remaining parameters in a three-step routine to minimize the distance between the models in the data and the model. In the first step, we use a minimization algorithm to jointly find 14 parameters governing the productivity process $(\overline{\mu}_u^1, \overline{\mu}_u^2, \underline{\mu}_u^1, \rho_u^1, \rho_u^2, \underline{\sigma}_u, \rho_z, \sigma_z)$, the non-homotheticities of top firms (κ_k^1, κ_l^1), and firms access to finance, $(\lambda_a, \lambda_v, \chi_0, \chi_1)$. For each combination of these 14 parameters, in the second step, we set four technology parameters $\{(\theta^h, \alpha^h)\}_{h=1}^2$ to directly math four moments: the capital-output ratio and labor share of top and bottom firms at age 19-20. We target these moments using the expressions in

[31]For example, we use the same definition of top 1 and bottom 99 firms percent firms at age 20 as in the data. Thus, while the top and bottom classifications are highly correlated with the $h = 1$ and $h = 2$ technologies, the mapping is not direct due to compositional changes in the size distribution.

Table 4: Targeted Moments

	Data	Model
(a) Cross-sectional and Longitudinal Moments		
Exit rate, all	0.08	0.08
Exit rate, Top-1 at age 0-2	0.08	0.05
Exit rate, Top-1 at age 19-20	0.03	0.04
Exit rate, Bot-99 at age 0-2	0.12	0.13
Exit rate, Bot-99 at age 19-20	0.06	0.06
Output share, Top-1	0.49	0.64
Output share, Top-10	0.75	0.71
Log output growth, Mean	0.06	0.05
Log output growth, Std. Dev.	0.18	0.16
Log output, Autocorr. 1-year	0.91	0.88
Log output, Autocorr. 3-year	0.84	0.81
Log output, Autocorr. 5-year	0.79	0.76
(b) Life Cycle of Top and Bottom Firms		
$y/\bar{y}_{0\text{-}4}$, Top-1 and Bot-99 at age 20	Figure 7, panel (a)	
k/y, Top-1 and Bot-99 at age 20	Figure 7, panel (b)	
wl/y, Top-1 and Bot-99 at age 20	Figure 7, panel (c)	
π/y, Top-1 and Bot-99 at age 20	Figure 7, panel (d)	
b/k avg., Top-1 at age 20	0.42	0.48
b/k avg., Bot-99 at age 20	0.33	0.32
$\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 0-4, Top-1 at age 20	0.05	0.05
$\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 19-20, Top-1 at age 20	0.00	0.00
$\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 0-4, Bot-99 at age 20	0.02	0.03
$\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 19-20, Bot-99 at age 20	0.01	0.00

Notes: Data moments from Spain. Model moments are computed from model simulated data following the same empirical strategy as in Section 3. Top-x and Bot-99 denote the top x and bottom 99 percent of the firm size distribution, respectively. Unless an age group is specified, firms are ranked across the entire distribution. $y/\bar{y}_{0\text{-}2}$ is output relative to average entrant. k/y is the capital-output ratio. wl/y is the labor share. π/y is the profit share. b/k is leverage, where debt in the model is $b = k - a$. $\mathbb{1}(\tilde{d}<0)|\tilde{d}|/k$ denotes equity injections relative to capital and is equal to zero if the firm pays dividends ($\tilde{d}>0$).

(9) and (10), where we allow for some dampening to account for the potentially higher capital costs and the presence of the capital and labor non-homothetic terms. Lastly, the third step chooses the three parameters governing the distribution of potential entrants, $(\mathcal{E}_1, \iota_0^1, \iota_0^2)$, to minimize the distance between the targeted moments in data and model.

Given our model's characteristics, it is not possible to directly match all parameters to specific moments. Yet, in what follows, we briefly describe which moments are most informative for each group of parameters. First, within the productivity parameters, those governing the ex-ante TFP process $(\{\overline{\mu}_u^h, \underline{\mu}_u^h, \rho_u^h\}_{h=1}^2, \underline{\sigma}_u)$ are primarily informed by

Table 5: Calibrated Parameters

		$h=1$	$h=2$	Description
Productivity	$\overline{\mu}_u^h - \underline{\mu}_u^h$	1.622	0.085	Ex-ante TFP growth u
	$\underline{\mu}_u^h$	0.399	0	Initial ex-ante TFP u
	ρ_u^h	0.970	0.976	Persistence, ex-ante TFP u
	$\underline{\sigma}_u$	0.031		Std. Deviation, initial TFP u
	ρ_z	0.784		Persistence, ex-post shock z
	σ_z	0.029		Std. Deviation, ex-post shock z
Technology	σ	1		CES capital-labor
	θ^h	0.758	0.920	DRS
	α^h	0.245	0.250	Capital share
	κ_k^h	15.51	0	Capital non-homotheticity
	κ_l^h	0.654	0	Labor non-homotheticity
Entrants	$\mathcal{E}_h$	0.016	1-$\mathcal{E}_1$	Share of h-type entrants
	ι_0^h	0.200	0.216	Asset distribution at entry
Finance	λ_a	1.435		Collateral constraint
	λ_v	0.040		Forward-looking constraint
	χ_0	0.721		Equity cost, fixed
	χ_1	0.070		Equity cost, linear

the dynamics of output over the life cycle of top and bottom firms (Panel (a), Figure 7). Notably, the initial differences in firm size between top and bottom firms are crucial to identifying the importance of ex-ante heterogeneity across firms.[32] The parameters for the ex-post shocks, (ρ_z, σ_z), are mainly determined by output cross-sectional and longitudinal moments, such as the average and dispersion of output growth and the autocorrelations of log output (Panel (a), Table 4).

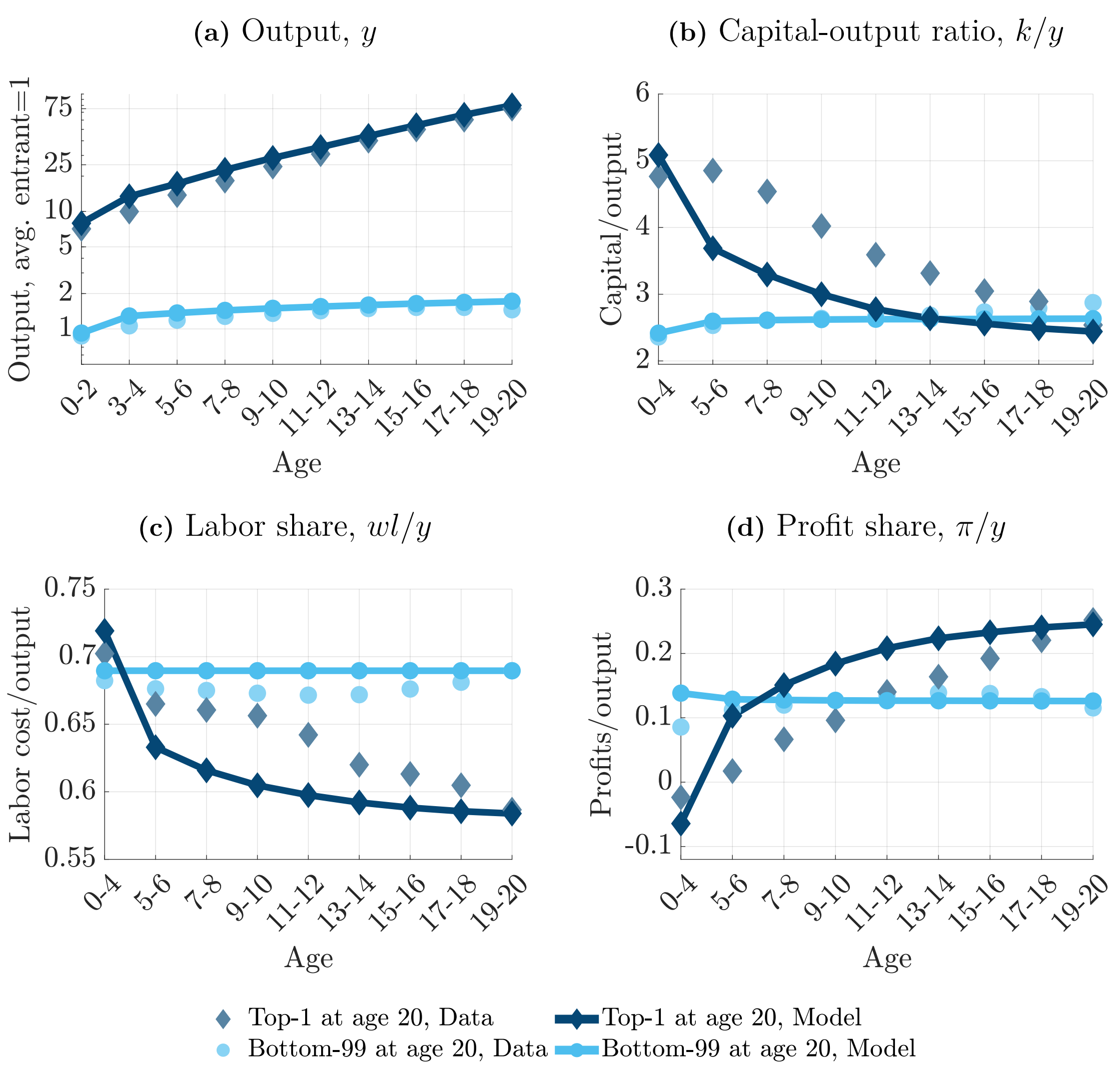
The technology parameters are mostly informed by our novel life cycle facts on top and bottom firms' input usage. As described above, the parameters governing the degree of DRS and the capital share, $\{(\theta^h, \alpha^h)\}_{h=1}^2$, are directly chosen to match four moments of top and bottom firms input usage at age 19-20. The non-homothetic terms for the top technologies, (κ_k^1, κ_l^1), are mostly informed by the life cycle path of the capital-output ratio, labor share, and profit share of the top 1 percent of firms at age 20 (Panels (b), (c), and (d) of Figure 7, respectively). The parameters governing the distribution of entrants, $(\mathcal{E}_1, \iota_0^1, \iota_0^2)$, are informed by both cross-sectional moments on the size distribution, such as the top 1 percent share of output, as well moments directly related to entrants exit rates

[32] In Appendix B.4, we provide further intuition on how the initial level and growth profile of top and bottom firms inform the ex-ante heterogeneity.

and initial growth (Panel (a), Table 4). Lastly, the parameters governing the degree of financial frictions in the model, $(\lambda_a, \lambda_v, \chi_0, \chi_1)$, are pinned down by the moments on the use of debt and equity financing of top and bottom firms (Panel (b), Table 4).

Model Fit Table 4 shows that our model closely aligns with the observed cross-sectional and longitudinal moments and the life cycle trajectories of top and bottom firms as Figure 7 illustrates. While the dynamics of top firms in our model are somewhat faster, the panels in this figure show that our model correctly matches the trajectories of top and bottom firms' output, input usage, and profits.

Figure 7: Life Cycle of Top and Bottom Firms in Data and Model



Notes: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1). Model moments are computed from model simulated data. The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms.

What explains these patterns? Figure B.2 illustrates the key ingredients driving the life cycle profiles of top and bottom firms in the model. Panels (a) and (b) show that, in the absence of input non-homotheticities, top firms' capital-output ratio and labor share would be mostly flat over their life cycle, which would be at odds with the data. On the other hand, panel (c) shows that, in the absence of financing frictions, the capital-output ratio of bottom firms would be flat over the life cycle. In contrast, it is increasing in the data and the baseline model. Finally, panel (d) shows that financing frictions do not affect the life cycle profile of the labor share for bottom firms, as a consequence of capital-labor substitution being $\sigma = 1$. Thus, consistent with our qualitative analysis in Section 5, the quantitative model requires that top firms' technologies be non-homothetic and that capital usage be initially low due to financing frictions to match the life cycle profiles of input usage for top and bottom firms.

7 Macroeconomic Implications

This section examines the macroeconomic implications of top and bottom firm dynamics. In the first two exercises, we assess the macroeconomic importance of two key ingredients of the model: input-specific fixed costs and forward-looking financing. In the last two exercises, motivated by recent aggregate trends, we examine the aggregate implications of growth driven by technological composition improvements (e.g., through greater diffusion of ideas and lower adoption costs) and the effects of long-run changes in interest rates, highlighting the importance of accounting for top firms' dynamics.

7.1 How Important are Non-Homothetic Input Costs?

In our quantification, we show that non-homotheticities in capital and labor are crucial to capture the input usage of top firms at their origins and subsequent life cycle trajectories. So far, however, it is unclear how relevant they are for the aggregate.

One direct measure of the importance of non-homotheticities is simply to use the calibrated values of $\{\kappa_k^h\}_{h\in\mathcal{H}}$ and $\{\kappa_l^h\}_{h\in\mathcal{H}}$ and the distribution of firms, to compute $\sum_{h\in\mathcal{H}}\int \frac{R\kappa_k^h}{Y}\mathrm{d}\Omega^h$ and $\sum_{h\in\mathcal{H}}\int \frac{w\kappa_l^h}{Y}\mathrm{d}\Omega^h$, respectively. These computations measure how large input-specific fixed costs are in terms of aggregate output. Table 6 Panel (a) shows that the direct relevance of non-homotheticities in capital and labor are negligible, 0.3% and 0.2% of aggregate output, respectively. Intuitively, non-homotheticities are relevant for top firms in their early stages but become less important as they grow. Therefore, they represent a pretty modest share of aggregate output.

Alternatively, we can assess the relevance of non-homotheticities indirectly by setting $\{\kappa_k^h\}_{h\in\mathcal{H}} = \mathbf{0}$ and $\{\kappa_k^h, \kappa_l^h\}_{h\in\mathcal{H}} = \mathbf{0}$. Table 6, Panel (b), presents the results. When shutting down only capital non-homotheticity, we find a remarkable change in aggregate output, which increases by 68%. Although capital non-homotheticities are negligible and

Table 6: Aggregate Importance of Non-Homotheticities

	Baseline	$\{\kappa_k^h\}_h = \mathbf{0}$	$\{\kappa_k^h, \kappa_l^h\}_h = \mathbf{0}$
(a) Direct			
$100\% \times \sum_{h\in\mathcal{H}} \int \frac{R\kappa_k^h}{Y} \mathrm{d}\Omega^h$	0.3%	0.0%	0.0%
$100\% \times \sum_{h\in\mathcal{H}} \int \frac{w\kappa_l^h}{Y} \mathrm{d}\Omega^h$	0.2%	0.5%	0.0%
(b) Indirect (Baseline = 1)			
Output, Y	1.00	1.68	1.69
Capital, K	1.00	1.55	1.56
Labor, L	1.00	1.36	1.35
Profits, Π	1.00	2.07	2.12
Wage, w	1.00	1.16	1.16
Mass $h = 1$ incumbents	1.00	3.61	3.61
Mass $h = 2$ incumbents	1.00	0.46	0.46

Notes: Panel (a) quantifies the direct aggregate importance of non-homotheticities based on the size of input-specific fixed costs relative to aggregate output. Panel (b) assesses the indirect importance of non-homotheticities, along with several other aggregate outcomes, by removing non-homotheticities from the model. In Panel (b), all aggregate values are normalized to the baseline model.

initially affect only a small subset of firms (i.e., measure $\mathcal{E}_1 = 0.016$), they prevent many top technologies from being financed, leading to their early exit and deterring their entry, which compounds into substantial aggregate effects. Indeed, the table shows that when non-homotheticities are absent, the measure of incumbent top technologies ($h = 1$) increases threefold. In general equilibrium, wages rise, inducing a significant share of bottom technologies ($h = 2$) to exit and deter their entry. This leads to high levels of firm concentration, a significant decline in the aggregate labor share despite wage increases, and a rise in aggregate profits. Lastly, we find that removing labor non-homotheticities, in addition to those in capital, does not lead to significant aggregate consequences.

In summary, while non-homothetic input costs are small relative to total output, they are very relevant, especially capital non-homotheticities, for aggregate outcomes when considering their indirect effects through the entry and exit of top technologies. Unlike a static perspective, these results highlight that what happens in the early days of top technologies—specifically those that start large, have high growth potential, and require substantial initial input usage—can compound into significant aggregate effects, even if their direct measurement seems negligible.

7.2 The Role of Forward-Looking Financing

In the following experiment, we study the aggregate implications of forward-looking financing. Firms in the model finance production through retained earnings (savings) and external funds via debt or equity. Debt financing consists of a standard backward-looking component based on the firm's assets and a forward-looking component, which depends on the current and expected trajectory of the firm's profits—which vary significantly across technology types. Moreover, equity financing is inherently forward-looking, as it also depends on the firm's value. To assess the role of forward-looking financing, we analyze forward-looking debt and equity financing separately and then jointly and compare them to the case of backward-looking debt financing.

Table 7 presents the implications of forward-looking financing for aggregate outcomes. All columns are normalized relative to the baseline model. The second column reports the case where firms cannot issue equity ($\chi_0 \to \infty$), but can use both types of debt. The third column shuts down forward-looking debt financing by setting $\lambda_v = 0$, restricting the collateral constraint to be purely asset-based. In both cases, we find significant reductions in aggregate output of 7% and 6%, respectively. Since top technologies ($h = 1$) have backloaded profits and rely much more on forward-looking financing, removing this source of funds reduces the measure of top incumbents by 18% and 35%, respectively. Conversely, bottom technologies ($h = 2$), which use much less forward-looking financing, benefit from lower wages in general equilibrium. This allows them to enter and survive at higher rates, significantly increasing their total measure. These compositional changes lead to lower aggregate profits relative to labor and capital. Overall, even when an alternative source of forward-looking financing is available, the aggregate impact of losing either equity or forward-looking debt financing is substantial, particularly for top firms that rely on external funds backed by their future profitability.

To assess the role of both sources of forward-looking financing, we study the case where firms cannot use equity or forward-looking debt financing, meaning they can only borrow against their assets ($\lambda_v = 0$, $\chi_0 \to \infty$). We contrast this with the case in the fifth column, where firms cannot borrow against their assets, a backward-looking source of external financing, but can access forward-looking debt and equity ($\lambda_a = 1$). In the absence of both sources of forward-looking funds, we find that aggregate output is 19% lower, while in the absence of backward-looking debt financing, output is 10% lower. These results underscore the importance of forward-looking financing, which is almost twice as important as the standard backward-looking debt. Notably, the compositional changes are more pronounced in the absence of forward-looking financing, with a 50% reduction in the measure of top technologies compared to a 21% reduction in the case without backward-looking debt. Interestingly, in the case without forward-looking financing, the measure of bottom firms increases by only 1%, despite significantly lower wages. Thus, bottom firms also rely on forward-looking financing, albeit to a lesser extent than that of top firms.

Table 7: Aggregate Implications of Forward-Looking Financing

	Baseline	No Equity $\chi_0 \to \infty$	No FL-b $\lambda_v = 0$	No FL-Fin. $\lambda_v = 0$, $\chi_0 \to \infty$	No BL-b $\lambda_a = 1$
Relative to Baseline (=1)					
Output, Y	1.00	0.93	0.94	0.81	0.90
Capital, K	1.00	0.93	0.95	0.82	0.84
Labor, L	1.00	0.97	0.98	0.89	0.94
Profits, Π	1.00	0.88	0.83	0.71	0.90
Wage, w	1.00	0.98	0.98	0.94	0.97
Leverage, B/K	1.00	0.99	0.84	0.84	0.24
Equity X/K	1.00	0.00	1.63	0.00	1.56
Incumbents, $h = 1$	1.00	0.82	0.65	0.50	0.79
Incumbents, $h = 2$	1.00	1.35	1.17	1.01	1.08

Notes: "No Equity" refers to the counterfactual where equity financing is unavailable. "No FL-b" corresponds to the counterfactual where $\lambda_v = 0$. "No FL-Fin." represents the case where neither type of forward looking financing is available (equity and forward-looking debt). "No BL-b" denotes the case where with no backward-looking debt, $\lambda_a = 1$, and hence firms cannot borrow against their assets. All values are normalized relative to the baseline model.

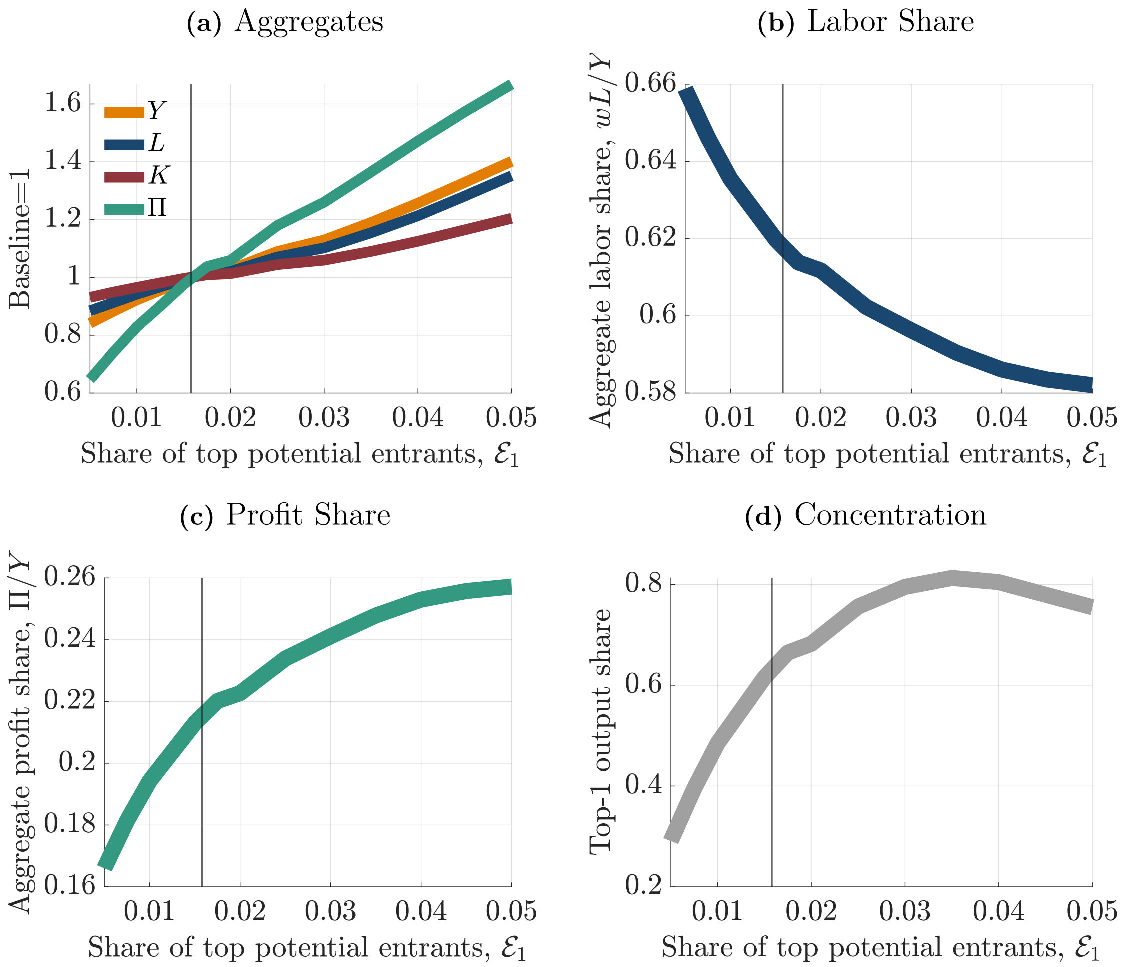
In sum, our results show that forward-looking debt and equity financing are important, especially for top firms. Intuitively, when top firms have access to forward-looking financing, they are close to being unconstrained despite requiring large initial inputs due to non-homotheticities, as they can finance themselves based on their high valuation (driven by expected high future profits). However, when this source of financing is unavailable, their low initial profits and high upfront costs due to non-homotheticities hinder their growth, deter entry, and accelerate their early exit, significantly reducing their prevalence in the economy and, consequently, aggregate economic activity.

7.3 Technological Frontier Changes

Next, we analyze how exogenous changes to the technological frontier, i.e., the distribution of $\{\mathcal{E}_h\}_{h \in \mathcal{H}}$ measures, affect aggregate outcomes such as the aggregate labor share and firm concentration. A greater availability of top technologies can be linked, for example, to better diffusion of ideas, both of which have been argued to be key drivers of economic growth (Lucas and Moll, 2014; Perla and Tonetti, 2014). In our experiment, we keep $\mathcal{E}$ fixed and vary the measure of potential entrants that are top technologies, $\mathcal{E}_1$, such that the measure of potential entrants with bottom technologies equals $\mathcal{E}_2 = \mathcal{E} - \mathcal{E}_1$.

Figure 8 Panel (a) shows how aggregate output changes with the availability of top technologies in the economy, i.e., as $\mathcal{E}_1$ varies. As expected, greater availability of high-growing technologies translates into greater aggregate output, capital, labor, and profits. Due to the change in the technology mix, specifically a higher prevalence of top tech-

Figure 8: Technological Frontier Changes



Notes: Steady state comparisons in GE solving for new equilibrium wage w. Aggregate variables in Panel (a) are normalized relative to the baseline economy.

nologies, labor doesn't increment as much as output, which, in Panel (b), translates into a lower labor share as the availability of top technologies increases.[33] In line with this pattern, in Panel (c), we observe that the profit share increments.

Finally, Panel (d) shows that output concentration at the top increases with the availability of top technologies; however, this relationship is non-monotone. If top technologies become more common, up to a certain point, several of them will populate the bottom 99 percent of the distribution, limiting how much concentration can increase as they become more prevalent. Thus, if an economy grows because of the higher availability of top technologies, our model predicts increased concentration in economic activity, lower labor shares, and higher profit shares.

[33]As argued for the U.S. in recent decades by Autor et al. (2020), Kehrig and Vincent (2021), and Hubmer and Restrepo (2022), the decline in the aggregate labor share in our experiment is also driven by the reallocation of economic activity to top firms, rather than by an increase in market power.

7.4 Long-Run Consequences of Interest Rate Changes

Finally, motivated by secular changes in the risk-free rate over the past decades (Bernanke, 2005; Summers, 2014), we study the long-run aggregate implications of real interest rate changes in an economy with top firms. To do this, we compare the effects of an interest rate increase in our baseline economy to those in an alternative economy calibrated to match the life cycle of the average firm. We consider the case of an interest rate increase, as the real rate in our baseline calibration is already low (2%). Table 8 presents the results for both economies: one calibrated for top and bottom firms and the other calibrated to match the average firm.

For both economies, in the second column, we perform an exercise where the r increases by 1.5% percentage points, but there are no changes in the discount factor β. In our top-bottom firms model, we find that aggregate output drops by 3.2% relative to a 1.0% drop in the average firm model. Since we keep the discounting β fixed, interest rate increases can affect aggregate outcomes through two channels. On the one hand, a higher interest rate raises the cost of capital, as $R = r + \delta$, which can reduce aggregate output. On the other hand, a higher interest rate increases the return on the risk-free asset, providing stronger incentives for self-financing, which can reduce misallocation and boost aggregate output.

Interestingly, in the average firm model, these two forces nearly offset each other, resulting in a contraction of aggregate output, capital, and labor by around 1%. However, higher risk-free rates in the top-bottom firms model lead to larger contractions in aggregate outcomes—two to three times greater. Intuitively, the greater cost of capital in the presence of non-homothetic technologies is equivalent to an increase in the capital-specific fixed cost for top technologies, i.e., $R\kappa_k^h \uparrow$. Consequently, top technologies are more affected by interest rate increases, as shown by their lower measure in equilibrium.

Notice that an increase in the interest rate r may pass through to a decrease in firms' discount factor β, for example, through foreign ownership of domestic firms. The discount factor has an implicit interest rate given by $r_\beta = 1/\beta - 1$, which we assume increases by 1.5 percentage points, thereby reducing the discount factor ($\beta \downarrow$). The third column for each model in Table 8 reports the results for this exercise. We now find a sizable contraction in aggregate output in both cases, though it remains larger in the top-bottom firms model, where output drops by 17.1% compared to 14.4% in the average firm model.

A decrease in the discount factor reduces the incentive to self-finance, which is important for average and bottom firms, as it lowers firms' values, limiting how much top firms can borrow against their expected high profits. Thus, a reduction in the discount

Table 8: Long-Run Consequences of Interest Rate Increases

	Top-Bottom Firms Model			Average Firm Model		
	Baseline	$r\uparrow$	$r\uparrow,\beta\downarrow$	Baseline	$r\uparrow$	$r\uparrow,\beta\downarrow$
Baseline (=1)						
Output, Y	1.000	0.968	0.829	1.000	0.990	0.856
Capital, K	1.000	0.985	0.755	1.000	0.993	0.766
Labor, L	1.000	0.987	0.893	1.000	0.994	0.902
Profits, Π	1.000	0.814	0.754	1.000	0.817	0.832
Wage, w	1.000	0.993	0.948	1.000	0.996	0.949
Incumbents, $h=1$	1.000	0.905	0.690	-	-	-
Incumbents, $h=2$	1.000	1.031	0.922	-	-	-
Measure of incumbents	1.000	1.030	0.919	1.000	0.957	0.824

Notes: "Top-Bottom Firms Model" refers to the results for our baseline calibration that targets the life cycle of bottom and top firms. "Average Firm Model" corresponds to the model where we target the average firm life cycle. For each calibration, all variables are normalized relative to the baseline quantification. The interest rate increases ($r\uparrow$) from 2% to 3.5%, while the implicit discount rate decreases from r_β to $r_\beta+1.5\%$, where $r_\beta=1/\beta-1$, in both models, such that $\beta\downarrow$.

factor significantly affects both types of firms, leading to a decline in the measures of top and bottom firms in the baseline model and a reduction of firms in the average firm model.

Overall, we find that it is important to account for top firms when assessing the long-run aggregate consequences of interest rate changes, as they transmit effects through different channels—for example, forward-looking financing rather than self-financing—and their initial cost of operation can be influenced by input-specific fixed costs, a feature that is less important for average or bottom technologies.

8 Conclusions

This paper documents novel facts about the origins and first twenty-year trajectories of the top 1 and bottom 99 percent firms. Using longitudinal firm-level data, we uncover striking differences in life cycle growth, input usage, and profitability across the firm size distribution. Compared to the bottom 99 percent, firms in the top 1 percent at age 20 start eight times larger and grow six times more in their first two decades. In terms of inputs, they begin with high capital levels, yet their capital-output ratio sharply declines over this period. Similarly, their labor share falls significantly. In contrast, the firms in the bottom 99 percent at age 20 exhibit an increasing capital-output ratio and a relatively stable labor share over time. As a result, top firms' profit share is much more backloaded toward the second decade of their life cycle than for bottom firms. These facts remain robust across multiple checks and hold across different points of the firm size distribution, various production sectors, and several countries.

We show that a firm dynamics model with ex-ante heterogeneity, non-homothetic technologies, and forward-looking financing can account for these empirical patterns. The central feature of our model is that high-growth potential firms have high input-specific fixed costs, which creates non-homotheticities in input usage. As firms grow, these costs become small relative to their size, leading to a decline in input-to-output ratios over their life cycle. High-growth potential firms benefit the most from forward-looking financing. In contrast, low-growth potential firms are more financially constrained and slowly increase their capital-output ratio over time. Thus, our model captures the empirical patterns through high growth and non-homothetic input costs for the top 1 percent of firms and financial frictions for the bottom 99 percent.

The findings in this paper point to several promising directions for future research. First, our empirical and quantitative results underscore the need to better understand the sources of firm ex-ante heterogeneity. The fact that firms in the top 1 percent at age 20 are nearly 11 times more likely to be foreign-owned and almost three times more likely to have owners with multiple firms, compared to the bottom 99 percent, suggests that exploring the role of multinationals, business groups, and serial entrepreneurs in firm dynamics is a natural next step. On the theoretical side, further investigation into the microfoundations of top firms' input-specific fixed costs studied in this paper would be valuable, especially for performing normative and policy analysis.

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Online Appendix for The Origins of Top Firms

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Table of Contents

A Data Appendix

A.1 The Orbis Database

This appendix provides definitions and additional results regarding the Orbis database. We primarily focus on Spain, the country we use for our baseline results.

A.1.1 Variable Definitions and Measurement

Firm Age Following Gopinath et al. (2017), firm i's age at time t is defined as $age_{it} = t - \tau_{i0} + 1$, where τ_{i0} is the year of incorporation reported in Orbis. The plus one term accounts for incomplete reporting spells at entry. For some companies, the incorporation year in the data and the actual foundation year might differ because of changes in the company's legal name due to restructures or mergers and acquisitions (M&A). To account for this issue, we searched for the foundation year of all the top 1 percent firms at age 20 in our sample. Whenever possible, we retrieve this information from the company website. Otherwise, we use publicly available information on the internet. For the cases where the incorporation and foundation year do not coincide, we use the foundation year of the company to compute firm age.[34] For the case of foreign multinational companies, which are older in their country of origin, we measure firm age since entering into the Spanish market, which coincides with the year of incorporation in Orbis.

Wage bill Regarding firms' income statement variables, we measure firms' wage bill as

$$wl_{it} = \texttt{staf}_{it}$$

where $\texttt{staf}_{it}$ are total labor costs of firm i at year t, using Orbis acronyms. For what follows, we follow BvD acronyms to refer to the variables in Orbis Historical.

Output We measure firms' output as value-added using a comprehensive measure of costs. In detail, we measure the output of firm i at time i as the sum of EBITDA (earnings before interest, taxes, depreciation, and amortization), acronym `ebta`, and labor costs

$$\begin{aligned} y_{it} &= \texttt{ebta}_{it} + \texttt{staf}_{it} \\ &= (\texttt{oppl}_{it} + \texttt{depr}_{it}) + \texttt{staf}_{it} \\ &= (\texttt{opre}_{it} - \texttt{cost}_{it} - \texttt{oope}_{it}) + \texttt{depr}_{it} + \texttt{staf}_{it} \\ &= \underbrace{\texttt{opre}_{it}}_{\text{Operating revenue}} - \underbrace{(\texttt{cost}_{it} - \texttt{staf}_{it} - \texttt{depr}_{it}) - \texttt{oope}_{it}}_{\text{Costs net of labor and capital}} \end{aligned} \tag{12}$$

where, as the last expression in (12) shows, it can be easily shown using the definitions of the variables in Orbis that our measure of output is equal to the sum of operating

[34] For example, the year of incorporation of "TELEFONICA DE ESPAÑA SAU", the largest telecommunications company in Spain, is 1997, reflecting a restructuring and name change of the company operations in Spain. Yet, according to the company website, this company was founded in 1924.

revenue (`opre`) minus a comprehensive measure of costs excluding those related to labor and capital: costs of goods sold (`cost`) net of labor costs (`staf`) and capital depreciation (`depr`) plus other operating expenses (`oope`).[35]

Capital, Equity, and Debt It is important to note that capital and the rest of the balance sheet variables are reported at the end of each year. Hence, as in Hsieh and Klenow (2009), we measure the stock of the variable at t as the average between the end-of-year values of $t-1$ and t. For our baseline results, we follow Kochen (2023) and measure capital as equity plus net financial debt

$$k_{it} = a_{it} + b_{it}.$$

Using Orbis acronyms, equity is equal to

$$a_{it} = (0.5 + 0.5L) \times (\texttt{toas}_{it} - \texttt{culi}_{it} - \texttt{ncli}_{it})$$

where `toas` denotes total assets, `culi` is current liabilities, `ncli` is non-current liabilities. The term $(0.5 + 0.5L)$ indicates the average between current and previous year balance sheet variables, where L is the lag operator.

We measure firms' net financial debt as

$$b_{it} = (0.5 + 0.5L) \times (\texttt{loan}_{it} + \texttt{ltdb}_{it} - \texttt{cash}_{it})$$

wher `loan` is short-term financial debt (payable within a year), `ltdb` is long-term financial debt, and `cash` denotes the firm's cash and cash equivalents.

In addition to our baseline definition of capital, we also present results for capital measured by tangible fixed assets, $(0.5 + 0.5L) \times \texttt{tfas}_{it}$, and alternatively by the sum of tangible and intangible fixed assets, $(0.5 + 0.5L) \times (\texttt{tfas}_{it} + \texttt{ifas}_{it})$. As Appendix A.2 of Kochen (2023) shows, our baseline measure of capital is broadly equal to the sum of tangible and intangible assets plus inventories (`stok`).

Profits Our baseline analysis focuses on a model-consistent definition of economic profits. Noting that the firm's capital is the sum of equity plus debt, $k_{it} = a_{it} + b_{it}$, we can rewrite economic profits, defined in (5), as

$$\begin{aligned} \pi_{it} &= y_{it} - wl_{it} - Rk_{it} \\ &= y_{it} - wl_{it} - \delta k_{it} - rb_{it} - ra_{it} \end{aligned} \tag{13}$$

where the rental cost of capital is $R = r + \delta$. We take the definition of economic profits in (13) to the data by setting $\delta = 0.05$ and $r = 0.02$, as in the model, and then directly measuring output y_{it}, labor costs wl_{it}, capital k_{it}, and equity a_{it} as defined above. Lastly, we measure the cost of debt rb_{it} using net financial expenses: $-\texttt{fipl}_{it} = \texttt{fiex}_{it} - \texttt{fire}_{it}$,

[35] This definition of output coincides to that used in Boar, Gorea, and Midrigan (2023), with the exception that we do not subtract for taxes.

where $\texttt{fiex}$ are financial expenses and $\texttt{fire}$ financial revenue.

In addition to the analysis using economic profits, we present results for profits measured by net income of the year, available in the data and equal to

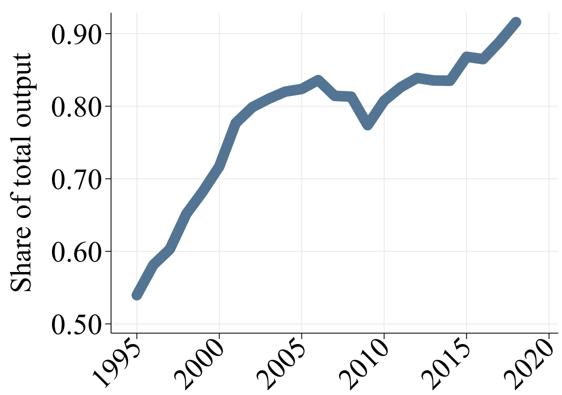
$$\begin{aligned}\pi_{it} &= \texttt{pl}_{it} \\ &= \texttt{plat}_{it} + \texttt{extr}_{it}\end{aligned}$$

where $\texttt{plat}$ is profits after taxation and $\texttt{extr}$ is extraordinary and other profit.

A.1.2 Coverage and Representativeness for Spain

Coverage To analyze the coverage of Orbis Historical, we follow Kalemli-Özcan et al. (2024) and compute the share of total revenue in our data relative to the *OECD STAN Database*. As we study the non-financial private sector, we focus on matched two-digit ISIC and year pairs in both Orbis and OECD STAN. Figure A.1 reports the share of total revenue (turnover) in our data relative to the official statistics in OECD STAN. The figure shows that coverage increases by year. The firms in our sample represent 54% of aggregate revenue in 1995 and 92% by 2018. On average, across all years, Orbis Historical covers 78% of total revenue in Spain, considering the matched two-digit sectors.

Figure A.1: Orbis Historical Share of Spain Total Revenue

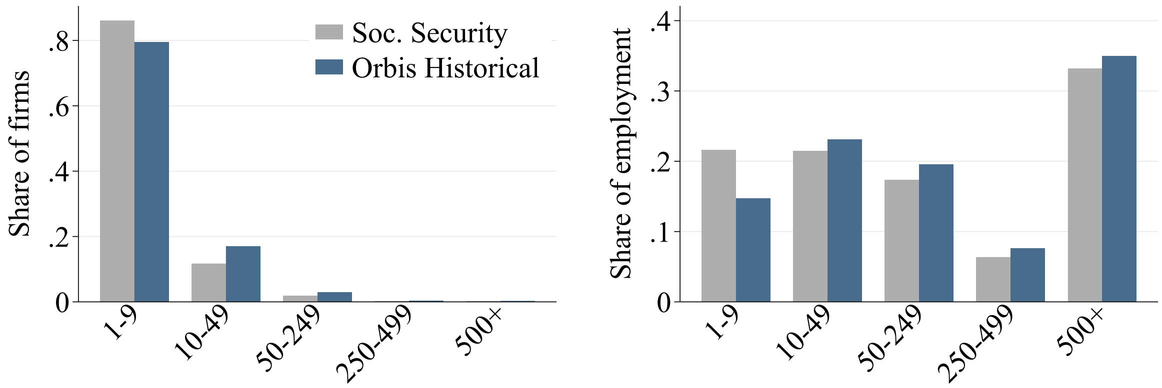


Notes: Share of total revenue relative to OECD STAN data for matched 2-digit sector-year pairs.

Size Distribution To test the representativeness of the data in terms of the size distribution, we compare the share of firms and share of total employment by firm size in Orbis Historical to official statistics from the Spanish Social Security (*Estadística de Empresas Inscritas en la Seguridad Social*). As before, we focus on employer firms. Panel (a) of Figure A.2 reports the share of observations by five different employment bins across these two datasets for 2019. Panel (b) presents the share of total employment. These two panels show that while Orbis Historical slightly underrepresents the smallest firms of 1 to 9 employees, it captures the full employment distribution quite well.

Figure A.2: Employment Distribution in Orbis Historical and Social Security Data

(a) Share of Observations **(b)** Share of Employment



Notes: Shares for Spain in the year 2019 considering employer firms. Soc. Security data comes from Estadística de Empresas Inscritas en la Seguridad Social published by the Spanish Labor Ministry.

A.2 Top Status Persistence

We now analyze the transitions of firms between the top and bottom of the size distribution depending on their age. We focus on firms in their early stages—ages 1 to 5—and at age 20, which is the maximum age considered in our baseline life-cycle analysis. Table A.1 presents the main results. We find that young top firms are much less likely to remain at the top many years later, even when conditional on survival, compared to top firms at age 20. The top firms between ages 1 and 5, conditional on surviving, have a 0.6 probability of remaining in the top 20 years later, while the probability of age-20 top firms is 0.8. Furthermore, we find that young top firms are much less representative, in terms of the number of firms and their output share, of top firms many years later than top firms at age 20. Top firms between ages 1 and 5 represent 25% of the firms and 65% of the output of top firms 20 years later, while top firms at age 20 represent 65% of the firms and 92% of the output, respectively.

These patterns suggest that there is substantial mobility between the top and bottom when firms are very young. In Section 3, we study whether the life cycle differs between firms that transition from the bottom to the top compared to those that remain at the top. Secondly, since top firms at age 20 are very likely to remain at the top and are quite representative of much older top firms, the early-life patterns of top firms at age 20, which are the ones we track back in our analysis of the origins of top firms in Section 3, are likely to be similar to those of much older top firms.[36]

[36]Notice that if there are strong cohort effects, older firms may have behaved differently. Then, our results are likely to be very informative of older top firms' early life cycle if we condition on cohort effects.

Table A.1: Top Transition and Representation

	Top at Age 1-5	Top at Age 20
P(Top at age $a+20$ | Top at age a)	0.62	0.81
Share of top firms at age $a+20$	0.25	0.65
Share of top firms' output at age $a+20$	0.51	0.92

Notes: The first line shows the probability that a top firm at age a is at the top of the entire distribution of firms 20 years later. The results are shown for top firms at ages 1 to 5 ($a \in \{1,2,3,4,5\}$) and at age 20 ($a = 20$) in columns one and two, respectively. The second and third lines show the share of firms and the output share within top firms at age $a+20$, respectively, that correspond to firms that were top at age a, where $a \in \{1,2,3,4,5\}$ and $a = 20$. To aggregate for ages 1 to 5, we take the average of the estimates.

A.3 Top Firms Pairs and Predictors

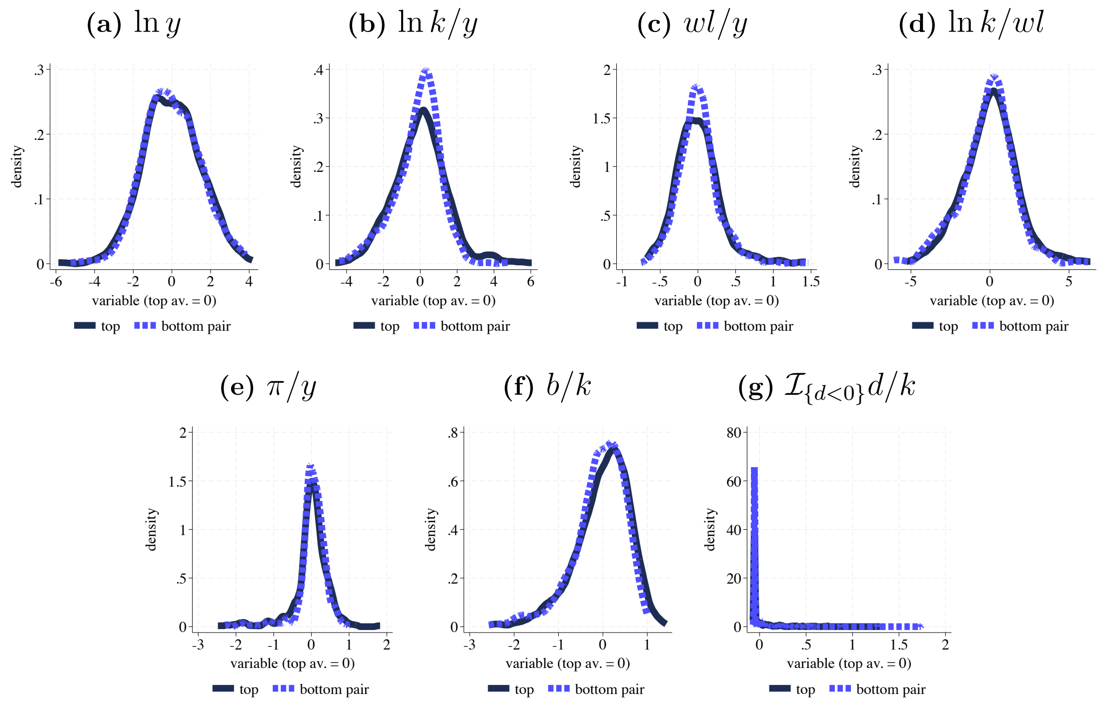
In this section, we study how firms that are similar to top firms in their initial years (i.e., bottom pairs) but end up in the bottom 20 years later evolve over their life cycle. Furthermore, we examine which observable characteristics predict whether firms are top at age 20.

Pairs Analysis Using a propensity score matching approach, we match each top firm with the closest bottom firm in its initial years. The variables used for matching are output level (y), input usage (k/y, wl/y, and k/wl), profitability (π/y), and financing (b/k, $\mathcal{I}_{\{d<0\}}d/k$), after removing sector and year fixed effects. We condition on bottom firms that survive to age 20 to perform the life cycle analysis. To obtain the propensity scores, we run a logit regression using these variables and then match each observation to its nearest neighbor, ensuring that propensity scores differ by no more than 0.05. This threshold is sufficiently loose that all top firms in this sample are matched, allowing us to retain a relatively large sample of more than 550 top firms. However, we find that 14.5% of the paired bottom firms are matched with more than one top firm, with a maximum of six matches for three of the paired bottom firms.

Figure A.3 shows the distribution of top firms and their bottom pairs are very similar across all the variables used for the matching procedure.[37] We estimate an empirical model based on (1) for the top and bottom pairs. Figure A.4 presents the results for output, input usage, and profits. Panel (a) shows that, as expected, the output growth of bottom firms that were similar to top firms at the start remains flat. Panels (b) and (c) show

[37]In the first step, we estimate a logit model, $\text{top}_i = \mathcal{L}(\beta \mathbf{x}_{i1} + \varepsilon_{it})$, where top_i indicates whether firm i is a top firm at age 20, and $\mathbf{x}_{i1}$ is a vector of characteristics of firm i at the first age group. Using the estimated probabilities, $\hat{p}_i = \mathcal{L}(\beta \mathbf{x}_{i0})$, we match top firms with bottom firms such that, for any top firm i, the bottom pair j is determined as $j = \arg\min_{\{j:\text{top}_j=0\}} |\hat{p}_i - \hat{p}_j| < 0.05$.

Figure A.3: Top and Bottom Pairs Matching Fit



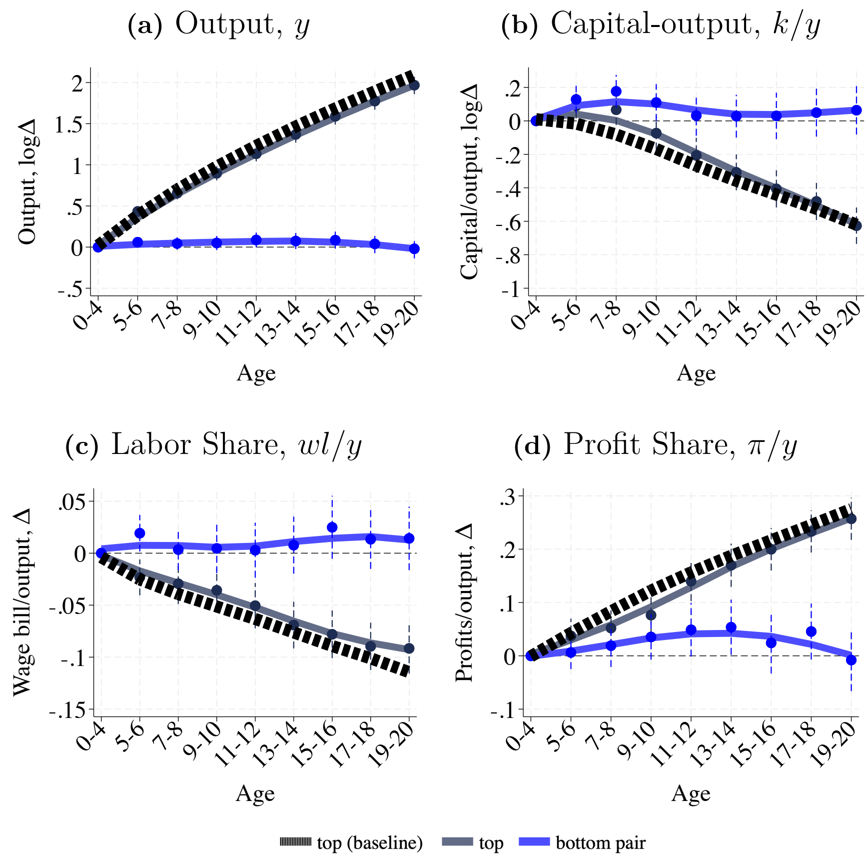
Notes: All variables exclude sector and year fixed effects, and are normalized such that the top firms' average is 0. The sample of top firms, is the one used for the matching. Details regarding the pairing are in the text.

that their capital-output ratio and labor share are roughly flat as well. Consequently, in panel (c), the profit share is also flat. Furthermore, the matched top firms behave very similarly to our baseline estimates. This is consistent with large initial shares being driven by input-specific fixed costs, which become relatively smaller as firms grow.

Predictors Next, we examine what initial observable characteristics predict whether a firm becomes a top firm at age 20. We use the same set of variables from the pair analysis as predictors, residualized by projecting them onto year and sector fixed effects. We estimate a probit model conditional on firms surviving until age 20 and another that includes firms that exit before age 20. Table A.2 shows the results. Firms with a larger initial size, a higher capital-output ratio, and greater equity injections are more likely to reach the top. On the other hand, the initial labor share, profitability, and leverage are not significant predictors. When including all variables simultaneously, we find that lower profitability and slightly lower leverage increase the likelihood of reaching the top at age 20.[38] These findings are overall consistent with our baseline exercise, even when including firms that exit before reaching age 20.

[38]Results remain unchanged if we use a linear probability model and/or raw variables with year and sector fixed effects.

Figure A.4: Life Cycle of Top Firms and Their Bottom Pairs



Notes:Life cycle trajectories of inputs and profit shares for the top 1% and their pairs in the bottom 99% of firms at age 20, estimated using (1), with the omitted age group being [0,4]. The regression for the output and capital-output ratio is in logs, while the labor and profit shares are in levels. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The thick black dashed line is the baseline profile for top firms. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

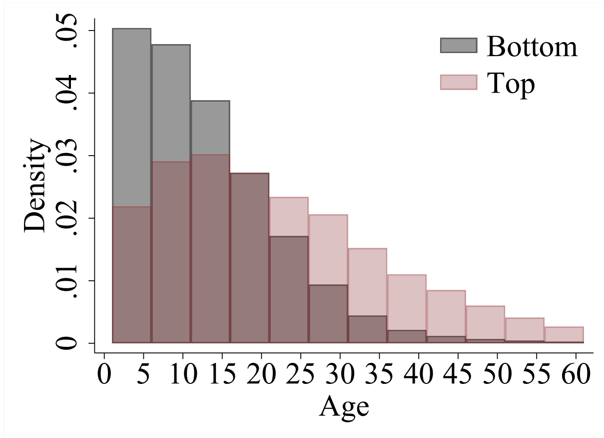
Table A.2: Predictors of Top Firms

	$\beta:\ \mathcal{P}(\beta\mathbf{x}_1) = \text{pr}[\text{top} = 1 \mid \mathbf{x}_1]$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) Conditional on Survival							
$\ln y$	0.599***						0.653***
	(0.00)						(0.000)
$\ln k/y$		0.057***					0.113***
		(0.00)					(0.000)
wl/y			0.035				0.167
			(0.53)				(0.191)
π/y				-0.043			-0.213**
				(0.34)			(0.01)
b/k					0.015		-0.081**
					(0.47)		(0.04)
$\mathbb{I}_{\{d<0\}}\lvert d\rvert/k$						0.449***	0.705***
						(0.00)	(0.00)
Observations	98,712	75,306	98,144	73,640	72,920	66,470	64,331
Pseudo R-2	0.32	0.00	0.00	0.00	0.00	0.00	0.36
(b) Including Exit							
$\ln y$	0.507***						0.550***
	(0.00)						(0.00)
$\ln k/y$		0.065***					0.150***
		(0.00)					(0.00)
wl/y			-0.065				0.173
			(0.19)				(0.11)
π/y				0.011			-0.097
				(0.80)			(0.17)
b/k					0.005		-0.137***
					(0.78)		(0.00)
$\mathbb{I}_{\{d<0\}}\lvert d\rvert/k$						0.411**	0.550**
						(0.00)	(0.00)
Observations	234,305	178,402	232,419	174,292	171,520	147,311	142,526
Pseudo R-2	0.28	0.01	0.00	0.00	0.00	0.00	0.31

Notes: The table shows the coefficients β and their p-values in parentheses for the probit model, where the outcome variable indicates whether the firm is top at age 20. The value of the predictors correspond to observable characteristics between age 0 and 4. Panel (a) conditions on firms that survive to age 20 and panel (b) includes firms that exit before age 20.

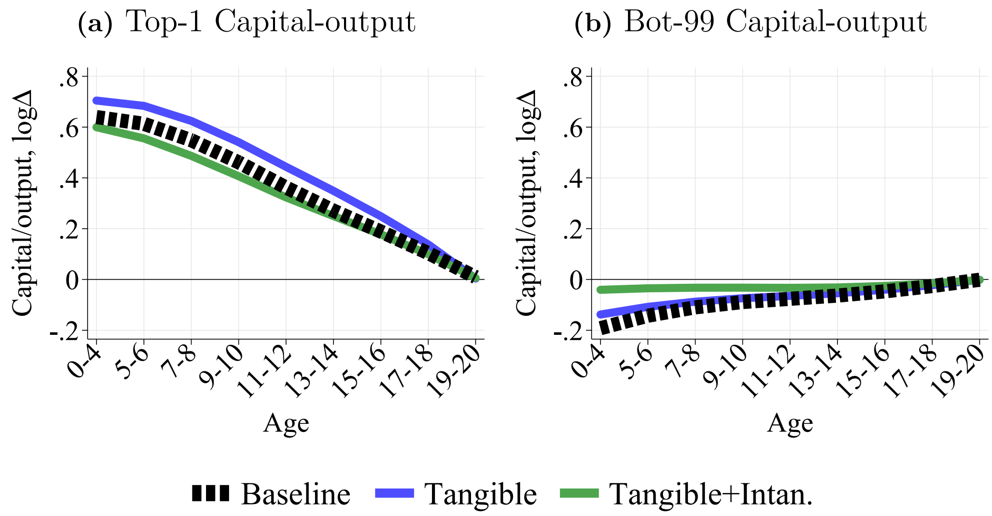
A.4 Additional Figures and Tables

Figure A.5: Age distribution: all and top firms



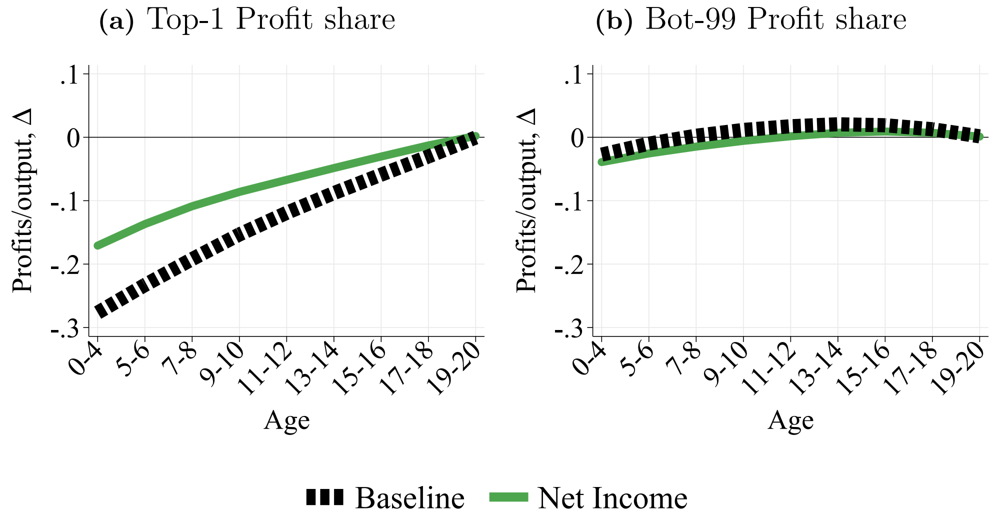
Notes: The figure shows the age distribution for all the firms and conditional on top firms.

Figure A.6: Alternative Measures for Firms' Capital
Life Cycle Changes in Capital-Output Ratio of Top and Bottom Firms



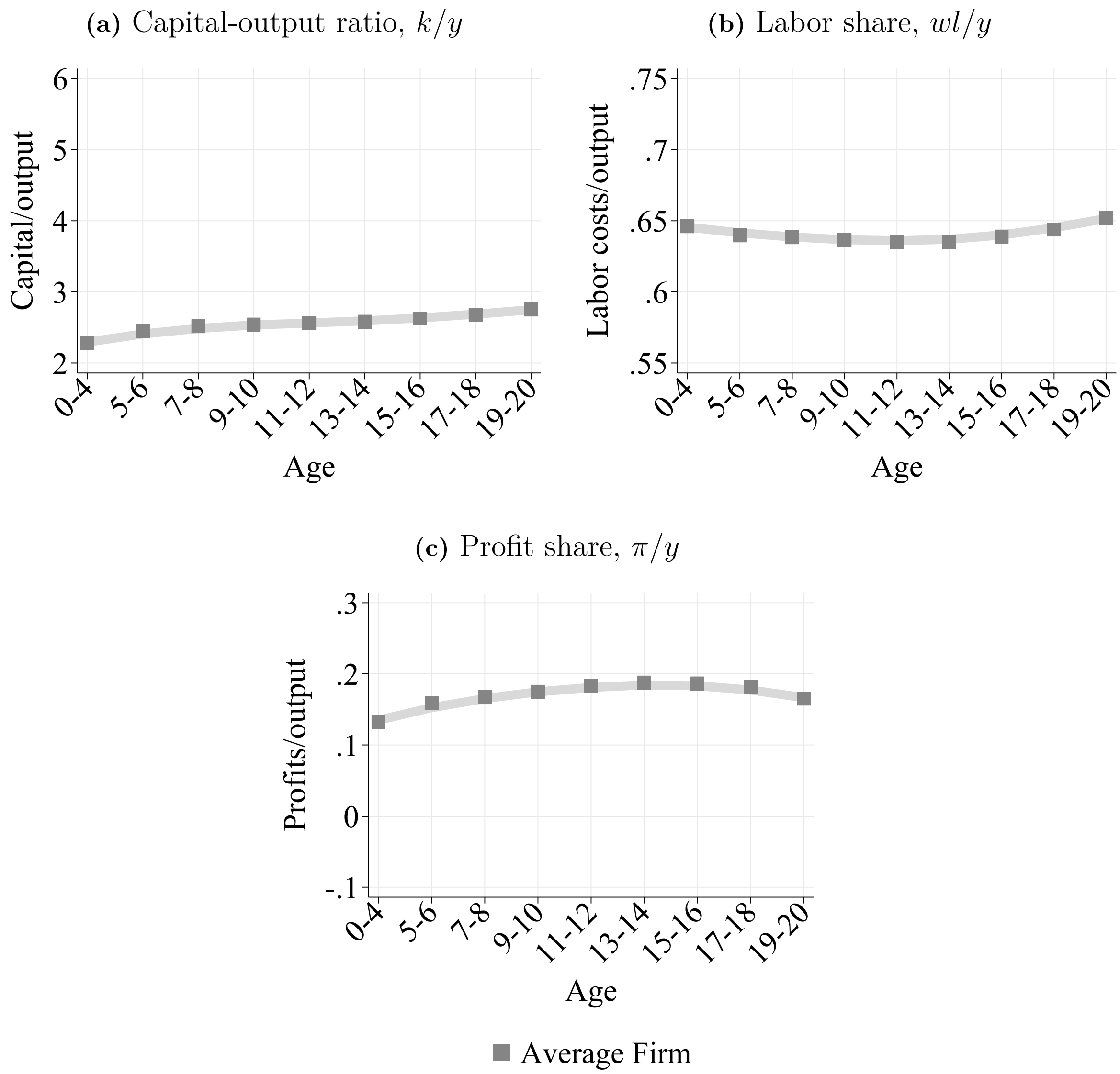
Notes: Life cycle changes for the capital-output ratio of top 1% and bottom 99% firms at age 20 considering two alternative measures of firms' capital. Changes are relative to the omitted group (age 19-20). The first alternative measure is tangible capital (`tfas`). The second measure is the sum of tangible and intangible capital (`tfas` + `ifas`). The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

Figure A.7: Alternative Measure for Firms' Profits
Life Cycle Changes in Profit Share of Top and Bottom Firms



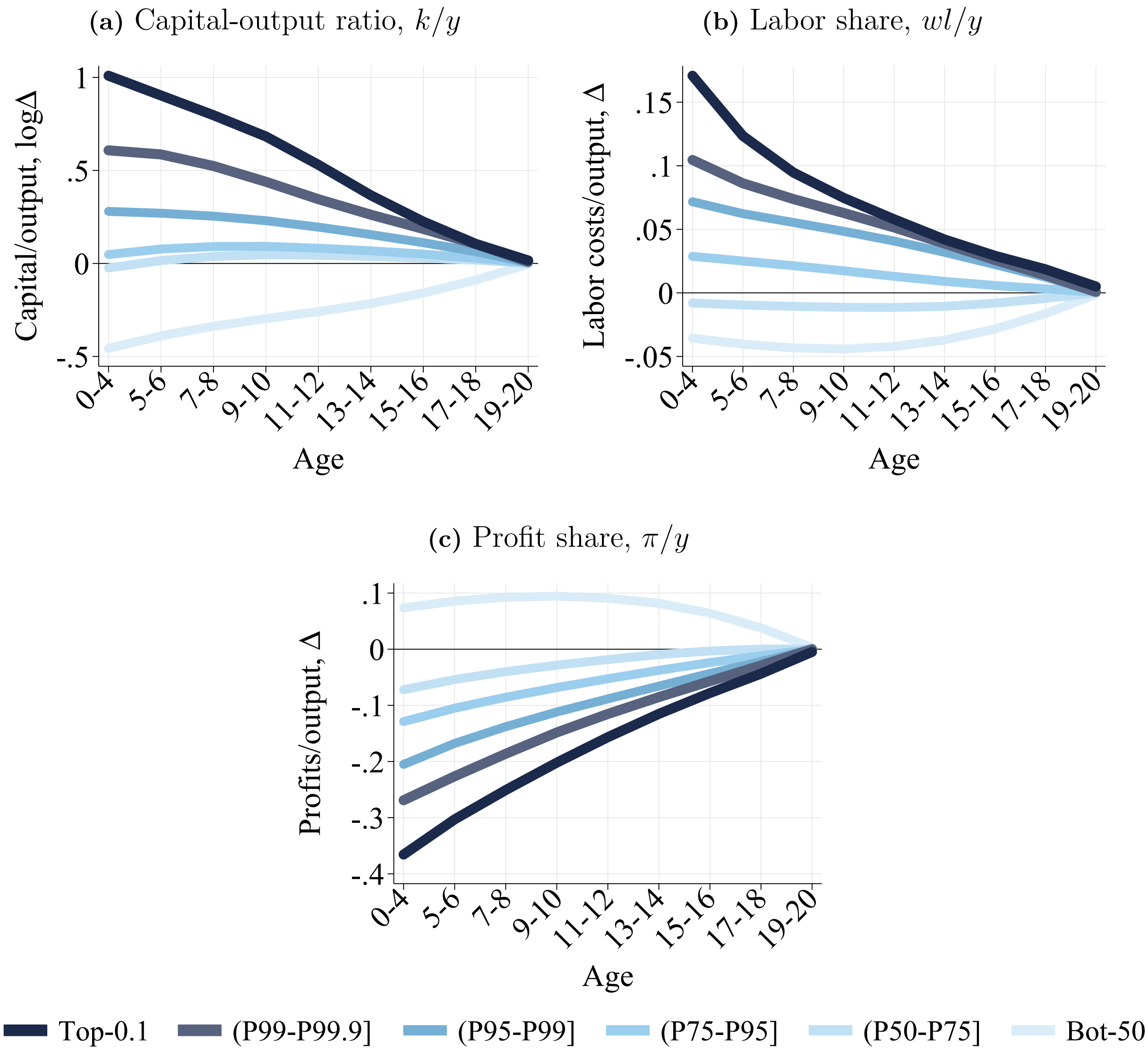
Notes: Life cycle changes for the profit share of top 1% and bottom 99% firms at age 20 considering firms' net income (variable `pl` in Orbis) as alternative measure for profits. Changes are relative to the omitted group (age 19-20). The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

Figure A.8: Inputs and Profit Share Over the Life Cycle of the Average Firm



Notes: Life cycle trajectories for the inputs and profit shares of the average firm estimated using a version of (1) putting together all the firms that survive to age 20. The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20). The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

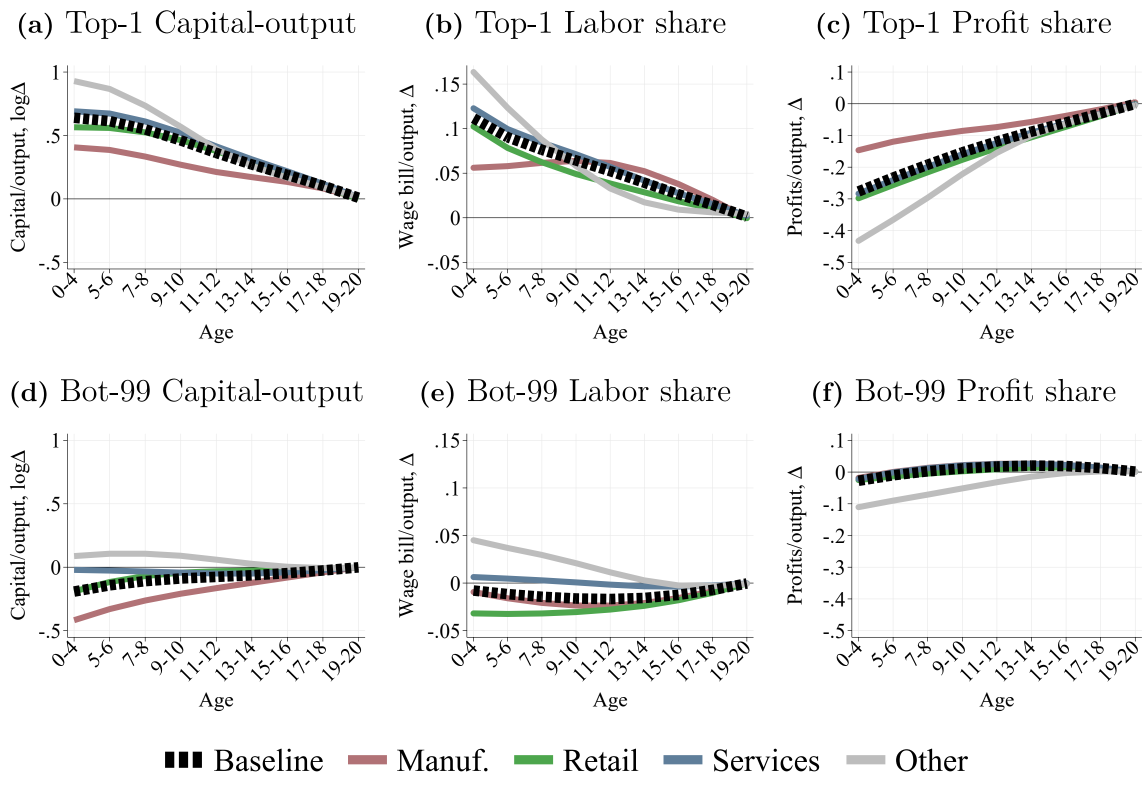
Figure A.9: Life Cycle Changes in Inputs and Profit Share by Size at Age 20



Notes: Life cycle changes for the inputs and profit shares of the top 0.01, percentile 99 to 99.9, percentile 95 to 99, percentile 75 to 95, percentile 50 to 75, and the bottom 50 percent firms at age 20. Life cycle changes are estimated using a version of regression (1) with these six groups of firms. Changes are relative to the omitted group (age 19-20). The lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

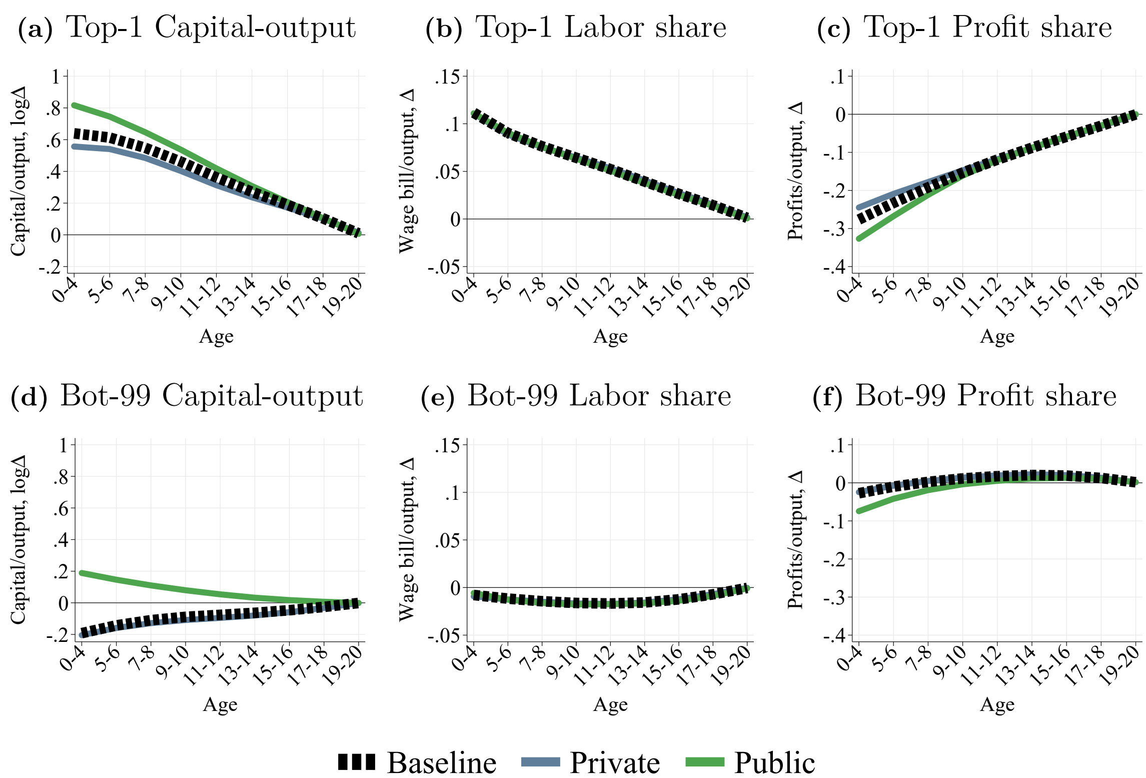
Figure A.10: Sectors

Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



Notes: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using a modified version of (1) with interactions by sector. Changes are relative to the omitted group (age 19-20). See Table 1 for the definition of the sectors. The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

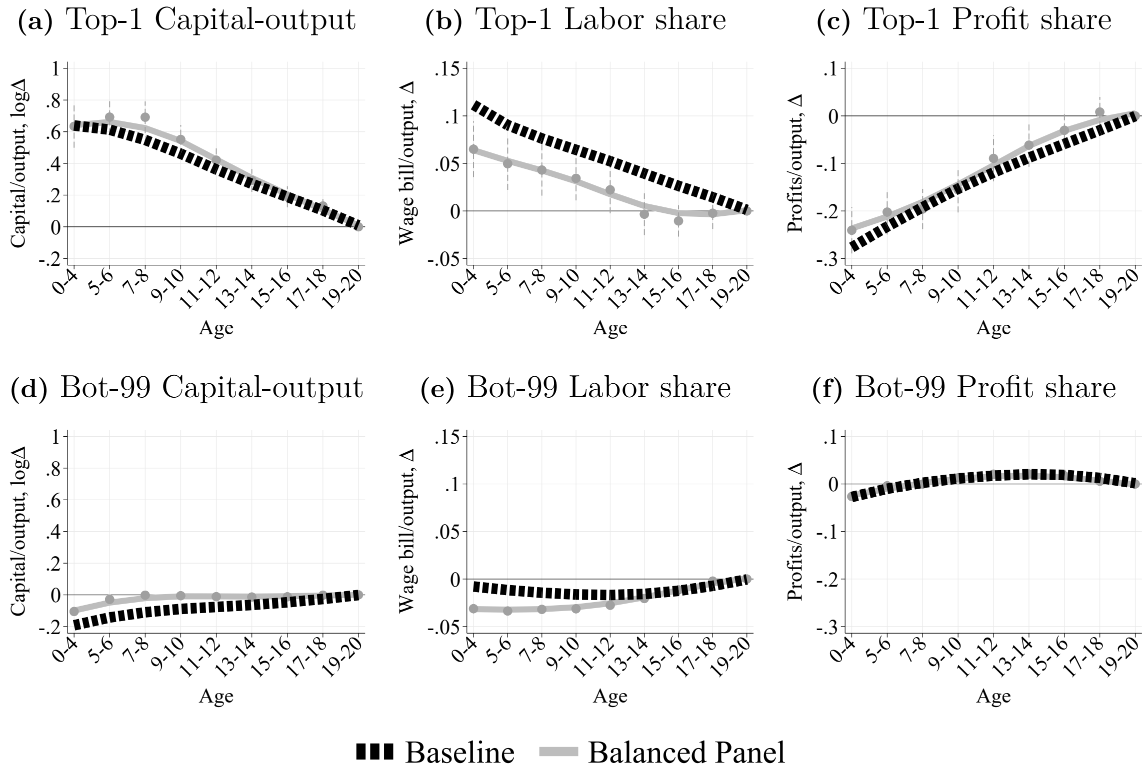
Figure A.11: Private and Public Firms
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



Notes: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using a modified version of (1) with a public firm dummy. Changes are relative to the omitted group (age 19-20). See Table 1 for the definition of the Public and Private firms. The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

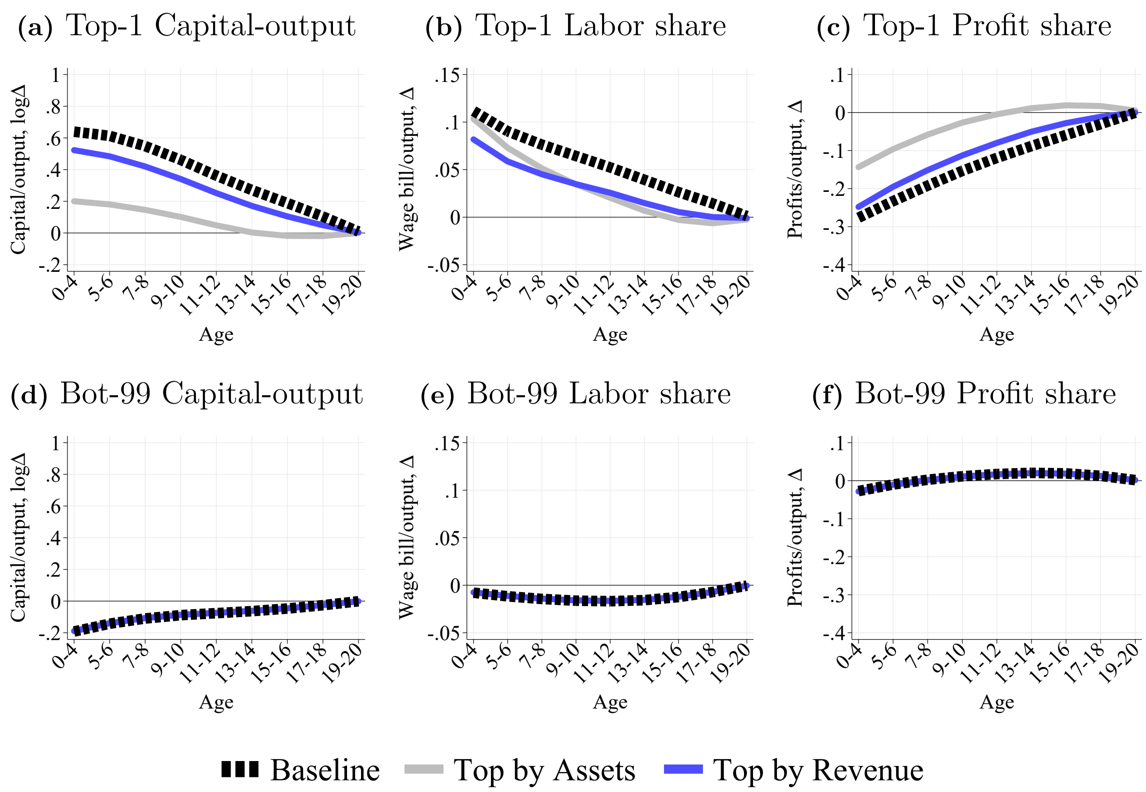
Figure A.12: Balanced Panel

Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



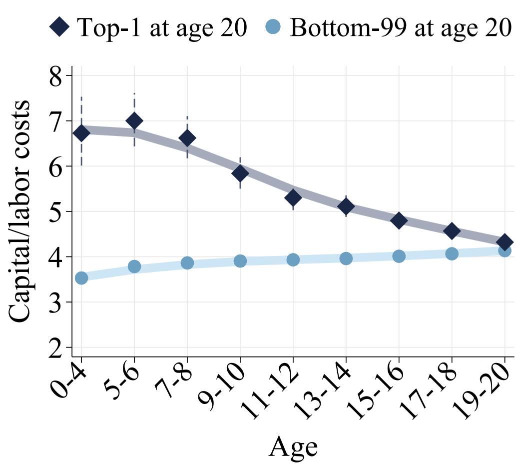
Notes: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1) for a balanced panel of firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors. The dashed dark line denotes the baseline estimation.

Figure A.13: Top and Bottom by Assets and Revenue
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



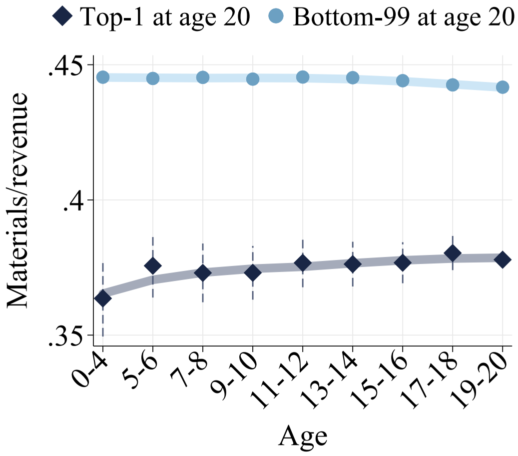
Notes: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1) for a balanced panel of firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors. The dashed dark line denotes the baseline estimation.

Figure A.14: Capital-to-Labor Costs Ratio of Top and Bottom Firms



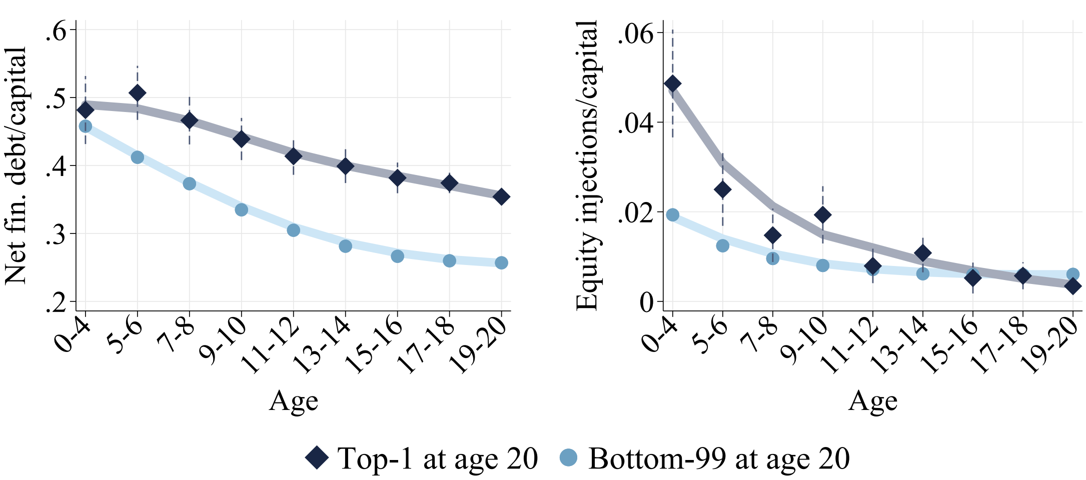
Notes: Life cycle trajectories of the capital-labor costs ratio of top 1 and bottom 99 percent firms at age 20 are estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines are smoothed scatterplots using locally weighted regressions.

Figure A.15: Materials-to-Revenue Ratio of Top and Bottom Firms



Notes: Life cycle trajectories of the materials-to-revenue ratio of top 1 and bottom 99 percent firms at age 20 estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

Figure A.16: Finance Over the Life Cycle of Top and Bottom Firms

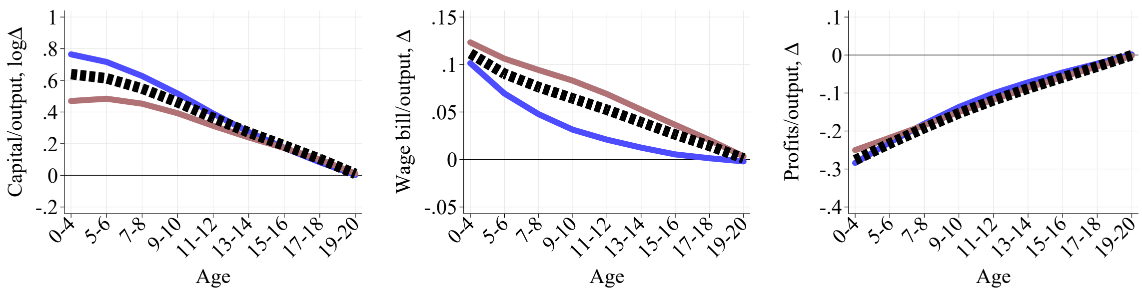


Notes: Life cycle trajectories for leverage and equity injections over capital for top 1 and bottom 99 percent firms at age 20 estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

Figure A.17: Top Firms by Owners' Characteristics

Life Cycle Changes in Inputs and Profit Share of Top Firms by Foreign Status

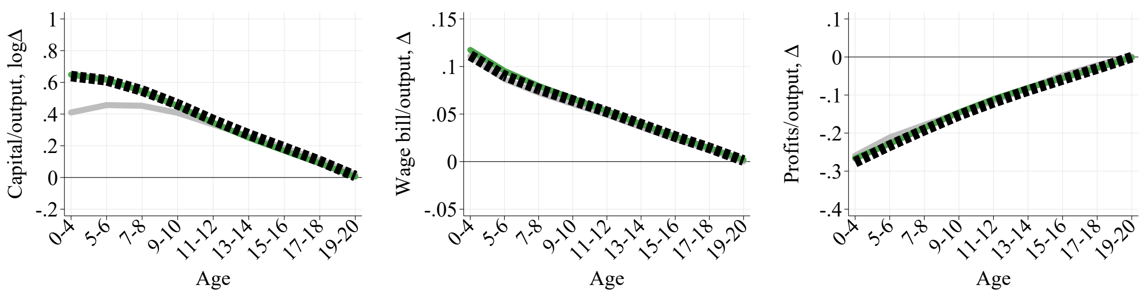
(a) Top-1 Capital-output **(b)** Top-1 Labor share **(c)** Top-1 Profit share



Baseline Foreign Domestic

Life Cycle Changes in Inputs and Profit Share of Top Firms by Multiple Firm Status

(d) Top-1 Capital-output **(e)** Top-1 Labor share **(f)** Top-1 Profit share



Baseline Single Firm Multiple Firms

Notes: Life cycle trajectories for the inputs and profit shares of the top 1 firms at age 20 estimated using a modified version of (1). The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed dark line denotes the baseline estimation.

B Model Appendix

B.1 Profit Maximization Problem

For ease of notation, we remove the time t and individual firm i subscripts.

Capital and Labor The FOC of problem (5) are

$$\theta^h (1-\alpha)^{\frac{1}{\sigma}} \frac{y}{\left(x^h\right)^{\frac{\sigma-1}{\sigma}}} \left(l-\kappa_l^h\right)^{-\frac{1}{\sigma}} = w$$

$$\theta^h \alpha^{\frac{1}{\sigma}} \frac{y}{\left(x^h\right)^{\frac{\sigma-1}{\sigma}}} \left(k-\kappa_k^h\right)^{-\frac{1}{\sigma}} = R+\mu,$$

where μ is the lagrange multiplier of the capital collateral constraint. Combining the FOC, we get that the optimal labor choice as a function of capita is

$$l = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\left(k-\kappa_k^h\right)+\kappa_l.$$

Using this, the input aggregator x^h can be written as

$$\begin{aligned} x^h &= \underbrace{\left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}_{A_k}\left(k-\kappa_k^h\right) \\ &= A_k\left(k-\kappa_k^h\right). \end{aligned}$$

Finally, using the FOC of capital we get the optimal capital choice as

$$k = \left[\frac{\theta^h \alpha^{\frac{1}{\sigma}} p^h\left(\mathbf{s}^h\right)}{A_k^{\frac{\sigma-1}{\sigma}-\theta^h}(R+\mu)}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h.$$

To find the *unconstrained* solution we set $\mu = 0$, then the optimal labor and capital choices are

$$l^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{w}\right)^{\sigma}\left(k^*-\kappa_k\right)+\kappa_l^h \tag{14}$$

$$k^* = \left[\frac{\theta^h \alpha^{\frac{1}{\sigma}} p^h\left(\mathbf{s}^h\right)}{\left(A_k^*\right)^{\frac{\sigma-1}{\sigma}-\theta^h} R}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h, \tag{15}$$

where $A_k^* = \left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{w}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. To solve for the *constrained* problem, first, we need to check if the solution is constrained and feasible. Thus, if $k^* < \bar{k}^h\left(a, s^h\right)$, then $k = k^*$; otherwise, $k = \bar{k}^h\left(a, s^h\right)$. Moreover, if $\bar{k}^h\left(a, s^h\right) > \kappa_k^h$, the solution is feasible; otherwise, we set the value of the firm to $-\infty$, forcing it to exit.[39] Under the constrained solution, we know that $\mu > 0$, which we find implicitly from

$$\bar{k}^h\left(a, \mathbf{s}^h\right) = \left[\frac{\theta^h \alpha^{\frac{1}{\sigma}} p^h\left(\mathbf{s}^h\right)}{A_k^{\frac{\sigma-1}{\sigma}-\theta^h}\left(R+\mu\right)}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h.$$

Once we find μ, we can compute the optimal solution as

$$l = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\left(\bar{k}^h\left(a, \mathbf{s}^h\right) - \kappa_k\right) + \kappa_l^h \tag{16}$$

$$k = \bar{k}^h\left(a, \mathbf{s}^h\right). \tag{17}$$

Input Usage We characterize now k/wl, wl/y, and k/y ratios. Combining the FOC we can find that

$$\frac{l-\kappa_l^h}{k-\kappa_k^h} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}$$

$$\frac{k}{wl} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w^{1-\frac{1}{\sigma}}}{R+\mu}\right)^{\sigma}\left(\frac{k/\left(k-\kappa_k^h\right)}{l/\left(l-\kappa_l^h\right)}\right). \tag{18}$$

Using the labor FOC and the relationship between capital labor we can rewrite the labor share as

$$\frac{wl}{y} = \theta^h\left(1-\alpha\right)\left[\left(1-\alpha\right) + \alpha\left(\frac{R+\mu}{w}\right)^{1-\sigma}\right]^{-1}\frac{l}{\left(l-\kappa_l^h\right)}. \tag{19}$$

Analogously, we can write the capital-output ratio as

$$\frac{k}{y} = \theta^h\alpha\left(\frac{1}{R+\mu}\right)\left[\alpha + \left(1-\alpha\right)\left(\frac{w}{R+\mu}\right)^{1-\sigma}\right]^{-1}\frac{k}{\left(k-\kappa_k^h\right)}. \tag{20}$$

B.2 Alternative Theory for Top Firms' Declining Input Shares

To simplify the notation we drop i firm and t time subscripts, and h technology-type superscripts. We assume that the firm doesn't face financial frictions and the technology is homothetic, but their price is elastic $p(y)$ to output and wages are elastic $w(l)$ to labor, and input efficiency can change over time z^j for $j = \{k, l\}$. Then, the firms solve the profit maximization problem

$$\max_{y,l,k} p\left(y\right)y - w\left(l\right)l - Rk - c_F \tag{21}$$

[39] An alternative assumption is that if the firm is unable to satisfy the capital non-homotheticity requirement, it remains dormant for that period and may produce later if it manages to obtain sufficient funds.

subject to

$$
\begin{aligned}
y &= p(\mathbf{s})\, x^{\theta} \\
x &= \left[\alpha^{\frac{1}{\sigma}}\left(z_k k\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left(z_l l\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.
\end{aligned}
$$

The FOC of problem (21) are

$$
\begin{aligned}
[l]: &\quad \left[\frac{\partial p(y)/p(y)}{\partial y/y}+1\right]\frac{\partial y}{\partial l}p(y) = \left[\frac{\partial w(l)/w(l)}{\partial l/l}+1\right]w(l) \\
[k]: &\quad \left[\frac{\partial p(y)/p(y)}{\partial y/y}+1\right]\frac{\partial y}{\partial k}p(y) = R.
\end{aligned}
$$

We denote $\mu_w = \frac{\partial w(l)/w(l)}{\partial l/l}+1$ as the proportional shadow cost of labor (i.e., if $\mu_w = 1$, there is no shadow cost), and $\frac{\partial p(y)/p(y)}{\partial y/y}+1 = \frac{1}{\mu_y}$, where μ_y represents the markup (i.e., if $\mu_y = 1$ there is no markup). Thus, the FOC can be rewritten as

$$
\begin{aligned}
\frac{1}{\mu_y}\theta(1-\alpha)^{\frac{1}{\sigma}} z_l \frac{(z_l l)^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}}} y p(y) &= \mu_w w(l) \\
\frac{1}{\mu_y}\theta\alpha^{\frac{1}{\sigma}} z_k \frac{(z_k k)^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}}} y p(y) &= R.
\end{aligned}
$$

Combining the FOC we get the relationship

$$
\frac{z_k k}{z_l l} = \frac{\alpha}{1-\alpha}\left(\frac{\mu_w w(l)}{R}\frac{z_k}{z_l}\right)^{\sigma}.
$$

Again doing some algebra and combining with the FOC we find

$$
\begin{aligned}
\frac{w(l)\, l}{p(y)\, y} &= \frac{1}{\mu_y}\theta(1-\alpha)^{\frac{1}{\sigma}}\left[\alpha^{\frac{1}{\sigma}}\left(\frac{z_k k}{z_l l}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\right]^{-1} - \frac{(\mu_w - 1)\, w(l)\, l}{p(y)\, y} \\
\frac{k}{p(y)\, y} &= \frac{1}{\mu_y}\theta\alpha^{\frac{1}{\sigma}}\frac{1}{R}\left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left(\frac{z_l l}{z_k k}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}.
\end{aligned}
$$

Combining the previous three equations, we get the wl/py, k/py, and k/wl ratios:

$$
\frac{w(l)\, l}{p(y)\, y} = \theta(1-\alpha)\frac{1}{\mu_y}\left[\alpha\left(\frac{R}{\mu_w w(l)}\right)^{1-\sigma}\left(\frac{z_l}{z_k}\right)^{1-\sigma} + (1-\alpha)\right]^{-1} - \frac{(\mu_w - 1)\, w(l)\, l}{p(y)\, y} \tag{22}
$$

$$
\frac{k}{p(y)\, y} = \frac{\theta\alpha}{R}\frac{1}{\mu^{y}}\left[\alpha + (1-\alpha)\left(\frac{\mu_w w(l)}{R}\right)^{1-\sigma}\left(\frac{z_k}{z_l}\right)^{1-\sigma}\right]^{-1} \tag{23}
$$

$$
\frac{k}{w(l)\, l} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{z_l}{z_k}\right)^{1-\sigma}\left(\frac{\mu_w}{R}\right)^{\sigma}(w(l))^{\sigma-1} \tag{24}
$$

B.3 Input Non-Homotheticity Microfoundations

In this section, we provide a microfoundation for the production technology with input non-homotheticities using homothetic technologies. Without loss of generality, we simplify the non-homothetic technology to be $y = p(\mathbf{s})(x - \kappa_x)^\theta$, where x input rental price is p_x, and there are no financing frictions. For this technology, we can solve analytically for the x/y ratio as:

$$\frac{x}{y} = \frac{\theta}{p_x}\left[1 + \frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}\right]. \tag{25}$$

Input Indivisibilities First, we can microfound this technology by assuming that firms produce using of two types of inputs x: a standard divisible input, $\tilde{x}$, and an indivisible input with a renting cost of $p_x\kappa_x$. The production technology is $y = p(\mathbf{s})\tilde{x}^\theta \mathbf{1}_x$ such that $\mathbf{1}_x = 1$ if the indivisible input was purchased. Thus, the firm profits are now

$$\pi = \max\left\{\underbrace{\max_{\tilde{x}} p(\mathbf{s})\tilde{x}^\theta - p_x(\tilde{x} + \kappa_x),}_{\text{rent indivisible input}} \underbrace{0}_{\text{not produce}}\right\}.$$

Thus, if the firm decides to produce, the optimal choice of $\tilde{x} = \left[\frac{\theta p(\mathbf{s})}{R}\right]^{\frac{1}{1-\theta}}$ and input-to-output ratio is simply $\frac{\tilde{x}}{y} = \frac{\theta}{p_x}$. However, if we redefine the input as $x = \tilde{x} + \kappa_x$, then doing some algebra we get

$$\frac{x}{y} = \frac{\theta}{p_x} + \frac{\kappa_x}{y} = \frac{\theta}{p_x}\left[1 + \frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}\right],$$

which is equivalent to (25).

Input-TFP Complementarities Next, we show how a technology featuring *learn-by-doing*, which decreases as firms grow, can serve as a microfoundation for the non-homothetic technology. Now we assume that the firm's TFP is $p(\mathbf{s}, x)$ and the production technology is $y = p(\mathbf{s}, x)x^\theta$, then the profits are

$$\pi = \max_x p(\mathbf{s}, x)x^\theta - p_x x$$

the FOC is

$$\frac{\partial p(\mathbf{s}, x)}{\partial x}x^\theta + \theta p(\mathbf{s}, x)x^{\theta-1} = p_x.$$

Note that the term $\frac{\partial p(\mathbf{s},x)}{\partial x} x^{\theta}$ captures additional marginal benefit if there are complementarities. Also, the input-to-output ratio is now

$$\frac{x}{y} = \frac{\theta}{p_x} + \frac{\epsilon_{p,x}}{p_x},$$

where $\epsilon_{p,x} = \frac{\partial p(\mathbf{s},x)/p(\mathbf{s},x)}{\partial x/x}$ is the elasiticity of the productivity to the input choice. We can interpret this elasticity as capturing potential *learn-by-doing*. Finally, if we parametrize this elasticity as $\epsilon_{p,x} = \theta \frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}$, such that the complementarities are lower as productivity grows, the input usage in equilibrium will be equivalent to (25).

B.4 Ex-Ante Heterogeneity and Top and Bottom Firms Growth

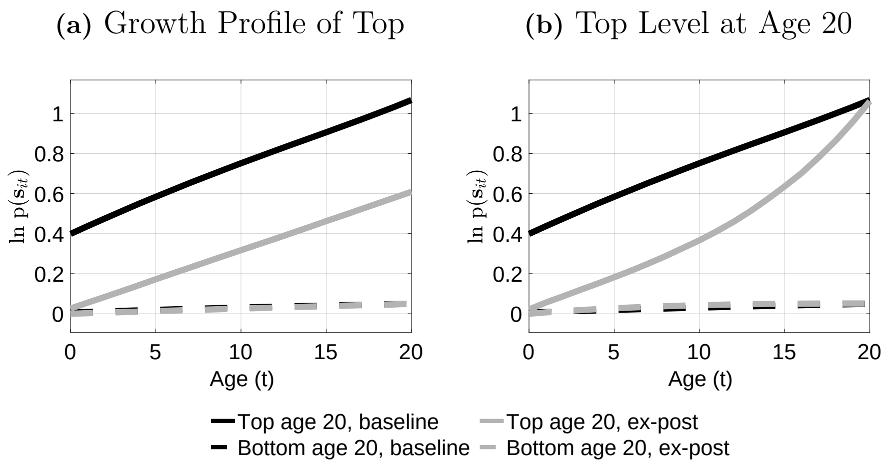
To provide intuition on how the initial level and growth profile of top and bottom firms inform the ex-ante heterogeneity component of our baseline TFP process, we present a simple example comparing a simplified version of our TFP process with a standard TFP process that includes only ex-post shocks. Figure B.1 shows the TFP life cycle path for our baseline TFP process without entry and exit selection and within type heterogeneity in the ex-post component ($\underline{\sigma}_u = 0$) and a TFP process without the ex-ante component, i.e., $p(\mathbf{s}_{it}) = \exp(z_{it})$ with $z_{it+1} = (1-\rho_z)\mu_z + \rho_z z_{it} + \varepsilon_{it+1}$. For the baseline TFP process we use the parameters of the TFP process and the measure of potential entrants $\{\mathcal{E}_1, \mathcal{E}_1\}$ calibrated in Section 6, to match the observed initial output level and growth profile of top and bottom firms.

Panel (a) compares the simplified baseline TFP process with a process that excludes the ex-ante component, calibrated to roughly match the life-cycle profile of bottom firms and the growth profile of top firms (i.e., slope) implied by our baseline process. To match the top firms' growth profile, we need an extremely persistent shock ($\rho_z = 0.99$) and very low volatility ($\sigma_z = 0.05$). Importantly, even if this unusual calibration allows us to match the growth profile of top firms, it remains far from fitting its level.

On the other hand, in Panel (b), we calibrate the ex-post process to fit the life-cycle profile of bottom firms and the TFP level of top firms at age 20 implied by our baseline process. This can be achieved with a relatively standard persistence ($\rho_z = 0.9$) and a relatively high volatility ($\sigma_z = 0.17$). In this case, the TFP process with only ex-post shocks implies an extremely high growth profile for top firms at age 20, as the initial shocks dissipate over time, causing the initial size of top firms at age 20 to be close to the average.

These examples illustrate that ex-ante heterogeneity in our model is informed by both the initial levels and growth profiles of top and bottom firms.

Figure B.1: TFP Process: Baseline vs. Ex-Post Heterogeneity

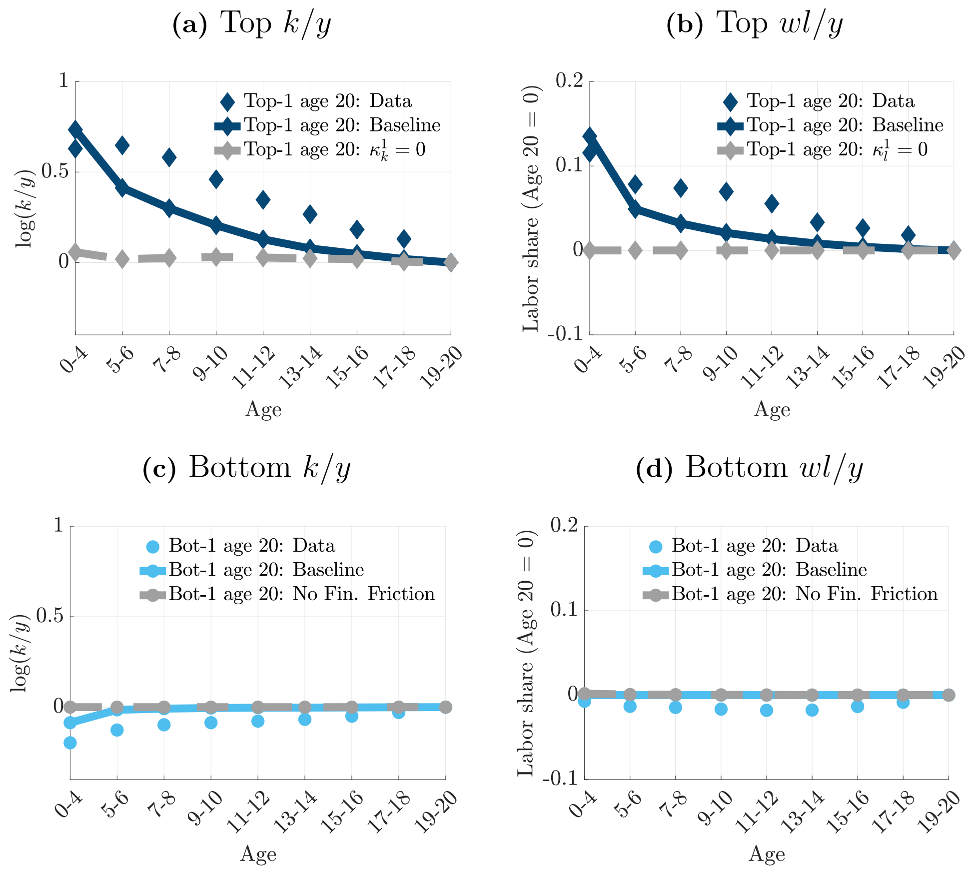


Notes: The figure shows the average life-cycle profile of total factor productivity (TFP) for top 1% and bottom 99% firms at age 20 in the baseline model–assuming entry and exit selection margins and no ex-ante within-type heterogeneity (i.e., $\underline{\sigma}_u = 0$)–with top and bottom firms represented by black solid and dashed lines, respectively. These profiles are compared to those obtained when the TFP process includes an ex-ante component $p(\mathbf{s}_{it}) = \exp(z_{it})$ with $z_{it+1} = (1-\rho_z)\mu_z + \rho_z z_{it} + \varepsilon_{it+1}$, shown by gray solid and dashed lines, respectively. Panel (a) shows the life-cycle profiles for the baseline TFP process and an alternative TFP process where the ex-post shocks have high persistence and low volatility $(\rho_z = 0.99, \sigma_z = 0.05, \mu_z = 0.3)$ such that it matches the bottom firms life cycle profile and growth profile of top firms (i.e., slope). Panel (b) shows the life-cycle profiles for the baseline TFP process and an alternative TFP process where the ex-post shocks have lowe persistence and high volatility $(\rho_z = 0.90, \sigma_z = 0.17, \mu_z = 0.07)$, such that it matches the bottom firms life cycle profile and the top firms' TFP level at age 20.

B.5 Input Usage Over The Life Cycle: Quantitative Model

In this section, we study the determinants of input usage profiles among top and bottom firms over their life cycle in the quantitative model. First we analyze how the life cycle input usage profile of top firms changes when input non-homotheticities are removed. Figure B.2, Panel (a) and (b), present the results, for reference we include the data and the baseline model patterns. We find that, in the absence of non-homotheticities, the input usage of top firms remains flat or slightly increases, contradicting the data, which show a sharp decline in input usage. Next, Panel (c) and (d) display the life cycle input usage patterns of bottom firms when financing frictions are removed. In this scenario, bottom firms' input usage remains roughly flat, as it is no longer distorted. This profile is consistent with the labor share pattern but contradicts the capital-output ratio pattern.

Figure B.2: Role of Non-Homotheticities and Financing Frictions



Notes: The figure shows the life cycle pattern of input usage, capital and labor, for top and bottom firms, respectively, in the data (dots) and using the model-simulated data. Panel (a) shows the top firms input usage when setting $\kappa_k^1 = 0$ and $\kappa_l^1 = 0$ (gray lines). The trajectory is in logs and is normalized to equal 0 at age 20. Panel (b) shows the bottom firms input usage when there are no financial frictions (gray lines). The trajectory is in percentage points and it is normalized to 0 at age 20.