

FIRMS' ROLLOVER RISK AND MACROECONOMIC DYNAMICS

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Job Market Paper

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Introduction

Motivation

- Last recessions have renewed the interest in role of financial factors in business cycles
- Potential failure of healthy firms has motivated unprecedented credit interventions
 - ▶ e.g., firm bailouts and direct lending to firms

This paper

- Macro consequences of **firms' rollover crises**
- Rollover crisis: economically solvent firm goes bankrupt because of debt rollover failure

Goal

- Quantitative macro model where rollover crises can be **identified** and **quantified**
- Implications of **firms' rollover risk** in macro dynamics and policy

What I Find

1. How **relevant** are firms' rollover crises in bankruptcy events?
 - + roughly **half** of bankruptcy events are driven by rollover crises
 - ▶ different bankruptcy choices (liquidation vs restructuring) key for identification
2. What are the **macroeconomic** implications during large recessions?
 - + rollover crises significantly **amplify** recessions
 - ▶ amplify depth and persistence → explain 10 to 30% output losses
3. What are the **policy** implications?
 - + credit policy **mitigates** rollover crises but subsidizes credit to weak firms
 - ▶ focus on direct-lending imperfectly-targeted credit policy

How I Do It

- GE model of heterogeneous firms with **default**
e.g., Khan Sengha Thomas 2016 Ottonello Winberry 2020
- Incorporate firms' **rollover crises** using tools from international macro literature
e.g., Cole Kehoe 2000 Bocola Dovis 2020
- Bankruptcy choices **informative** of rollover crises' incidence
Corp Law literature, e.g., Jackson 1986; and Corbae D'Erasmus 2021
- **Quantitative** analysis of U.S. economy
amplification of output losses and credit policy in recessions

Related Literature

- **Financial heterogeneity and default risk in macro**

Cooley Quadrini 2001 Hennessy Whited 2005 Cooley Marimon Quadrini 2004 Jermann Quadrini 2012
Arellano Bai Kehoe 2019 Khan Seng Thomas 2020 Ottonello Winberry 2020 Corbae D'Erasmus 2021

+ incorporate rollover crises jointly with bankruptcy procedure

- **Rollover crises in macro-finance**

- . banks: Gertler Kiyotaki 2015 Amador Bianchi 2021
- . sovereign debt: Cole Kehoe 2000 Bocola Dovis 2020 Aguiar Chatterjee Cole Stangebye 2021
- . corporate finance theory: Morris Shin 2004 He Xiong 2012a,b; Cheng Milbradt 2012 Zhong 2021
- . bankruptcy law: Jackson 1986 Ayotte Skeel 2013

+ quantification for firms using bankruptcy choices

- **Direct credit policy interventions in recessions**

Crouzet Tourre 2021 Elenev Landvoigt Van Nieuwerburgh 2021 Ebsim Faria-e-Castro Kozlowski 2021

+ policy works through insurance channel with rollover risk

Outline

- Macro Model of Firms' Rollover Crises
- Identification
- Macroeconomic Consequences

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- **Macro Model of Firms' Rollover Crises**
- Identification
- Macroeconomic Consequences

Overview of the Model

Environment

- GE framework populated by heterogeneous firms
- Firms use internal resources and borrow to finance investment and production
- Other agents: HHs, creditors, and capital producer

Three key ingredients

1. Endogenous default risk
2. Debt rollover crises
3. Heterogeneous firms and bankruptcy choices

Firms Objective

- Firm i objective is to max \mathbb{E} discounted value of dividends

$$V_{i0} = \sum_{t \geq 0} \mathbb{E}_0[\Lambda_t d_{it}]$$

with Λ_t HH's SDF and d_{it} firm's dividends

Technology

Firms hire l at wage w and use inherited k to produce unique final good with technology

$$f(z, \omega, k, l) = z (\omega k)^\alpha l^\nu$$

- DRS in (k, l) $\alpha + \nu < 1 \rightarrow$ optimal size
- persistent productivity shock $z = \rho_z z_{-1} + \epsilon_z$ where $\epsilon_z \sim \text{iid}$
- capital quality shock $\omega \sim \text{iid log-normal truncated for quant purposes}$
 - ▶ changes effective value of capital

Resources

Each period, firms inherit (k, b) and decide (k', b')

- Cash-on-hand

$$n = \underbrace{\max_l f(z, \omega, k, l) - wl}_{\text{operational profits}} + \underbrace{(1 - \delta)q\omega k}_{\text{selling value of capital}} - \underbrace{b}_{\text{maturing debt}}$$

- Flow of funds constraint

$$\underbrace{d}_{\text{dividends}} = \underbrace{n}_{\text{internal resources}} + \underbrace{Q(.)b'}_{\text{external resources}} - \underbrace{qk'}_{\text{capital purchases}} \geq 0$$

Debt Price and Sunspot Shock

- Debt price schedule

$$Q(z, n, \phi, k', b')$$

depends on current and future default decisions

- **Sunspot** shock
 1. firms draw $\phi \sim U[0, 1]$ iid across time and firms
 2. if $\phi < \eta$ then $Q = 0$ for firms exposed to rollover crises
 3. η common across firms

Bankruptcy

Emulating US Bankruptcy Code bankrupt firms can choose to

1. Liquidate and exit (Chapter 7)

- creditors recover $\alpha_7 \in [0, 1]$ of k and firm exits with value $V = 0$
- choice **after** issuing b' (Cole-Kehoe 2000)

2. Restructure liabilities and continue operating (Chapter 11)

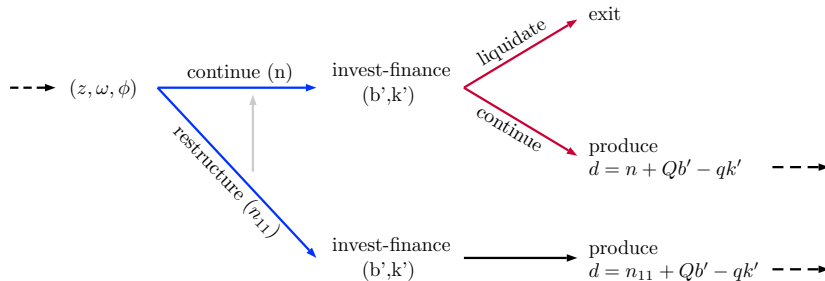
- internal resources if firm restructures

$$n_{11} = n + \underbrace{(1 - \alpha_{11})b}_{\text{benefit}} - \underbrace{c_{11}k}_{\text{cost}}$$

debt haircut $(1 - \alpha_{11}) \in [0, 1]$ endogenous **bargaining protocol** and cost $c_{11} \geq 0$

- **no rollover crises** in new debt issuance b'

Within Period Timing



- all uncertainty realized at the beginning of the period
- characterize firm choices backwards

note: timing for non-exit firms

(Exogenous) Entry and Exit

Technical and quantitative assumptions

- Exogenous exit probability γ (KST 2016, stationary dist)
 - ▶ if receive shock the firm exits after production
- Entrants enter on average productivity $m\%$ below ergodic distribution average (OW 2020, life-cycle firms)

[more details](#)

Multiple Equilibria

- Firm choose to liquidate if they can't satisfy $d \geq 0$
- Feedback between debt price and default choice of firms today

$$Q = \underbrace{\mathbb{I}_{d \geq 0}}_{\text{liquidation choice}} \underbrace{\tilde{Q}}_{\text{price if no liquidation}}$$

$$d = n - qk' + Qb' \geq 0$$

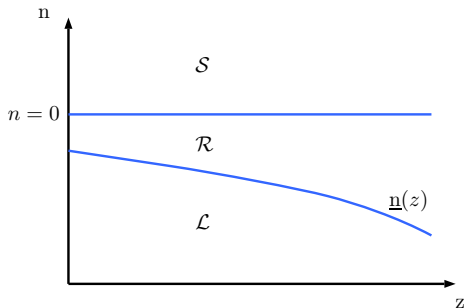
- Multiplicity if

$$Q = 0 \iff d < 0 \quad (\text{rollover crises})$$

$$Q > 0 \iff d \geq 0 \quad (\text{repay})$$

Liquidation

- **Fundamental** state-space (z, n) is divided in three regions



- \mathcal{S} : $Q = 0$ then continue if $d = n + \underbrace{\max_{k'} \{-q k'\}}_0 > 0$
- \mathcal{L} : $Q = \tilde{Q}$ then liquidate if $d = n + \underbrace{\max_{k', b'} \{-q k' + \tilde{Q} b'\}}_{-\underline{n}(z)} < 0$
- \mathcal{R} : liquidate if $Q = 0$, continue if $Q = \tilde{Q} > 0$

- **Rollover crisis** when firm is in \mathcal{R} (solvent) but $\phi < \eta$
- if $\eta \rightarrow 0$ then only fundamental defaults (e.g., KST, OW) rollover risk

Restructuring

Firms restructure their liabilities if

1. firm insolvent or with rollover crisis (key: outside option is to continue)
2. creditors recover more than in liquidation: $\alpha_{11} > R(\omega, b, k) \equiv \min \left\{ 1, \alpha_7 \frac{(1-\delta)q\omega k}{b} \right\}$
3. firm satisfies $d \geq 0$ after restructuring: $n_{11} \geq \underline{n}(z)$

Nash bargaining further details

Illustrative parametrizations

- $c_{11} \rightarrow \infty$: fundamental and non-fundamental liquidations
- $c_{11} = 0$ and $\alpha_{11} = 1$ (i.e., $n = n_{11}$): only fundamental liquidations

Q and restructuring

Firm's Problem

- $\tilde{V}(z, n)$ value of solvent firm without a rollover crisis today

$$\tilde{V}(z, n) = \max_{d, k', b'} d + \mathbb{E}_{(z' | z; \omega'; \phi')} [\wedge \{(1 - \gamma) \tilde{V}(s') + \gamma V_{\text{exit}}(s)\}]$$

subject to

$$d = n - qk' + \tilde{Q}(z, b', k') b' \geq 0$$

$$s' = (z', \omega', \phi', k', b')$$

where $\tilde{Q}(\cdot)$ fundamental debt price [details Q](#) and $V_{\text{exit}}(s)$ value of exiting firm [details exit](#)

- policy functions $k'(z, n)$ and $b'(z, n)$ solve dynamic problem

- $V(\cdot)$ before restructure choice for non-exiting firms

$$V(s) = [1 - 1_{\{\text{ch11}\}}(s) - 1_{\{\text{ch7}\}}(s)] \tilde{V}(z, n) + 1_{\{\text{ch11}\}}(s) \tilde{V}(z, n_{11})$$

where liquidation and restructuring choices are consistent with characterization

Other Agents and Equilibrium

1. Households detail

- ▶ labor supply
- ▶ Euler equation and SDF Λ

2. Capital producer detail

- ▶ sells capital at price q
- ▶ standard aggregate capital adjustment function

3. Creditors detail

- ▶ debt price Q through no-profit condition
- ▶ discount at SDF Λ

Equilibrium full definition distribution's law of motion

- steady state and transitions

Outline

- Macro Model of Firms' Rollover Crises
- **Identification**
- Macroeconomic Consequences

Identification

Questions

- How many firms are in \mathcal{R} ?
- Value of η ?

Steps

1. Calibration of **standard** parameters to match relevant moments of U.S. economy
2. Calibration of parameters related to **bankruptcy procedure**
 - ▶ validate with bankruptcy predictors and investment dynamics during recessions
3. Quantify η and share of exposed firms

Data sources

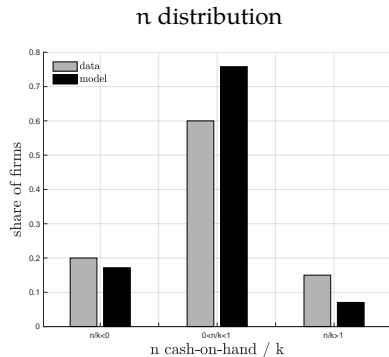
- NIPA, Compustat, Federal Judicial Center-IDB, LBD, other papers

Calibration Standard Parameters

	Parameter	Value	Calibration
<p>Calibration strategy</p> <ul style="list-style-type: none"> 9 fixed and 4 fitted parameters not related to rollover crises and bankruptcy parameters: preferences, technology, stochastic process, entry/exit fit moments: employment, investment, balance sheet, life-cycle 	<i>Fixed</i>		
	$\beta = 1/(1 + r)$	0.99	fixed to $r = 0.05$ annual
	Φ	1.16	fixed to match 58% emp rate
	ν	0.64	fixed labor share
	α	0.21	fixed capital share
	δ	0.025	fixed to match BEA quarterly
	ρ_z	0.90	fixed
	γ	0.02	fixed to exit rate w/o default
	ψ	2	agg AC fixed to lit standard
	b_0	0	fixed to no net debt entrants
	<i>Fitted</i>		
	σ_z	0.032	internally calib
	$\underline{\omega}$	-0.33	internally calib
	k_0	0.16	internally calib
	m	-0.24	internally calib

Relevant Moments

Moment	Data	Model
<i>Credit spreads</i>		
credit spread: $\mathbb{E}[r^Q - r]$	0.02	0.01
default rate: $\mathbb{E}[\mathbf{1}_{Ch7} + \mathbf{1}_{Ch11}]$	0.03	0.03
<i>Investment heterogeneity</i>		
avg invest rate: $\mathbb{E}[i/k]$	0.12	0.17
sd invest rate: $SD[i/k]$	0.34	0.36
<i>Life-cycle</i>		
share exit	0.10	0.11
(L age 1) / L	0.03	0.04
# firms age 1 / # firms	0.10	0.11
# firms age 2 / # firms	0.08	0.09
<i>Aggregates</i>		
K/Y	3.00	2.59
I/Y	0.17	0.15
gross debt: $\mathbb{E}[\mathbf{1}_{b>0}b]/Y$	1.05	1.79



measurement

Calibration Bankruptcy

- $(\alpha_7, \psi_{11}, c_{11})$ match debt haircut under Ch 11 and Ch 7, and leverage in Ch 11

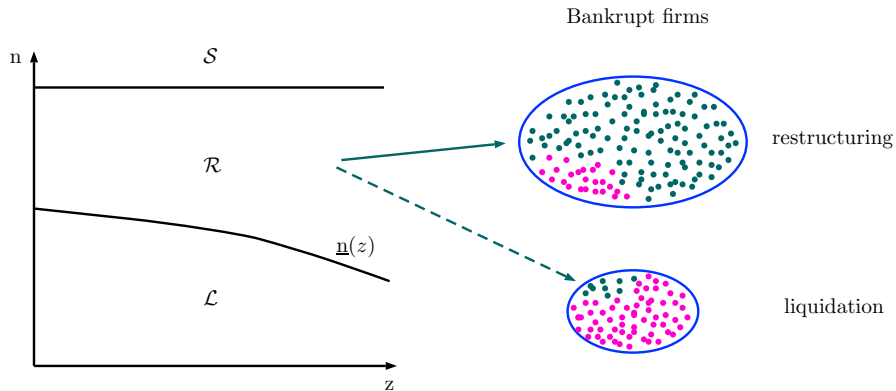
Param.	Value	Moment targeted	Data	Model
α_7	0.38	$\mathbb{E}[R_7]$	0.27	0.29
ψ_{11}	0.89	$\mathbb{E}[\alpha_{11}]$	0.69	0.82
c_{11}	0.40	$\mathbb{E}[b'/k' \mid \text{Ch 11}]$	0.73	0.67

Untargeted moments

- distribution of leverage in Ch 11 and predictors of Ch 11 [validation](#)
- investment heterogeneity in last recessions [details](#)

Identification of η Intuition

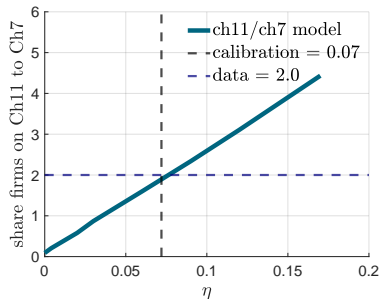
- with rollover crises



proposition

Incidence of Rollover Crises (1/2)

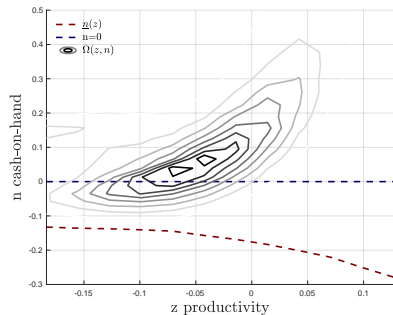
- higher η (in stst) then higher share of firms in Ch 11 (relative to Ch 7)



- $\eta \approx 0.07$

Incidence of Rollover Crises (2/2)

Steady state distribution $\Omega(z, n)$
before bankruptcy choice



$$\underbrace{\int_{(z,n) \in \mathcal{R}} d\Omega(z, n)}_{\substack{\text{share of firms exposed} \\ 0.20}} \times \underbrace{\eta}_{\substack{\text{crises likelihood} \\ 0.07}} = 1.5\%$$

Result I: roughly half of bankruptcy events are rollover crises

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- **Macroeconomic Consequences**

Macroeconomic Consequences

1. Recessions

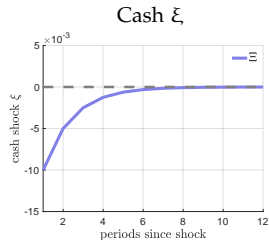
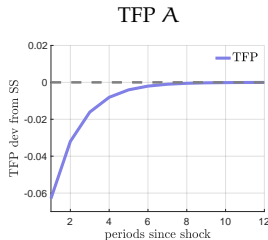
2. Policies

- perfect foresight path of unexpected shock

Recession Shocks

- Shocks unexpected and perfect foresight of path
- Temporary with persistence 0.5
- Definition of shocks:
 - ▶ TFP A: prod funct $Az f(k, \omega, l)$
 - ▶ Cash ξ : reduction in n by ξk

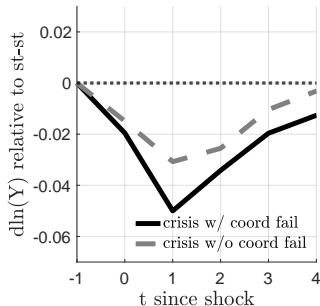
Shocks' path



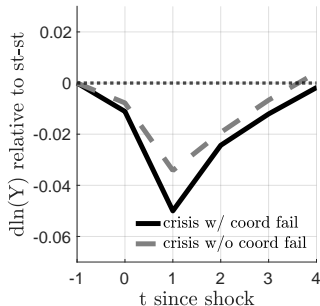
Study path of Y with and without rollover crises in the recession

Recessions and Rollover Crises

TFP A



Cash ξ



η shock st-st comparison

- Firm exit increases [details](#)

Result II: rollover crises explain 15% to 30% of output losses in recessions

Credit Policy Intervention

- **Direct lending** policy: external resources for firm eligible to the policy are

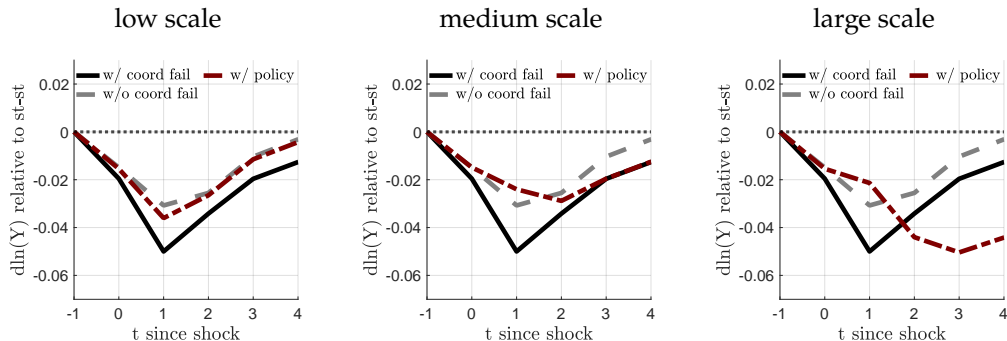
$$\max \left\{ \underset{\text{market}}{Q(s, b', k')}, \underset{\text{government}}{Q^g(.)} \right\} \times b'$$

- Policy **workings**: take eligible firm with $(z, n) \in \mathcal{R}$ under a rollover crisis
 - ▶ if $d = n + \max_{k', b'} \{-qk' + Q^g b'\} > 0 \Rightarrow$ **preclude crisis**
- Imperfect policy faces **trade-off** between precluding rollover crises and future debt overhang parametrization announcement and implementation direct lending vs credit guarantees

Study policy effectiveness in the recession

Credit Policy Quantification

- Policy active for first two periods and TFP shock driven crisis cash shock fiscal losses



Result III: imperfectly-targeted credit policy benefits are ambiguous

Concluding remarks

Concluding remarks

- Quantitative framework with heterogeneous firms and rollover crises
- Bankruptcy choices indicative of rollover crises incidence
- Results
 1. rollover crises are relevant way how firms fail
 2. rollover crises can significantly amplify recessions
 3. credit policy has ambiguous benefits

Future research avenues

- Extensions: (i) liability structure management (ii) heterogeneous investors
liab structure data ex-ante cost
- Empirical work
- Other applications (e.g., sovereign debt, financial firms)

Thank you!

Extra Slides

US Bankruptcy Code

Bankrupt firms use chapter 11 (11 U.S.C.) or 7 (7 U.S.C.) of US bankruptcy code

- Chapter 7
 - associated with firm's liquidation
 - case impartial trustee appointed to sell the bankrupt firms assets to pay creditors
- Chapter 11
 - associated with firm's restructure (or reorganization)
 - large firms also use to piecemeal liquidate the firm ("363 sale", 11 U.S.C. § 363(a))
 - debtor presents plan, and needs to be approved by judge and, ultimately, negotiated with and voted by creditors
 - provisions to preclude creditor's coordination problem, e.g.,
 1. automatic stay 11 U.S.C. § 362(a): prevents creditors demand payment
 2. debtor-in-possession protection 11 U.S.C. § 1101: allows new financing
 3. creating creditors' committees 11 U.S.C. § 341

Bankruptcy Procedure

- Only firms that are insolvent or under a rollover crises may restructure their debt
- Recovery rate $\alpha_{11}(\cdot)$ determined by

$$\alpha_{11}(z, k, b, \omega) = \arg \max_{\alpha_{11}} \left[\underset{\text{firm's surplus}}{V(z, n^{11}) - 0} \right]^{1-\Xi} \left[\underset{\text{creditor's surplus}}{\alpha_{11}b - R(k, b, \omega)b} \right]^{\Xi}$$

where $\Xi \in [0, 1]$ barg power of creditors, we need that $n_{11} > \underline{n}(z)$ and $\alpha_{11} > R(k, b, \omega) = \min \{1, \alpha_7 (1 - \delta) q \omega k / b\}$

- For computational reasons I approx the barg. Max recov rate $\{\alpha_{11}^{\max} : n_{11} = \underline{n}(z)\}$ and min recov rate $\alpha_{11}^{\min} = \alpha_7^{\min} = R(k, b, \omega)$, then recov rate linear comb of those rates with $\psi_{11} \in (0, 1)$ the weight to creditors

Entry and Exit

Exogenous exit

- Firms receive exog exit shock with prob γ
- Exiting firms allowed to restructure and liquidate before producing then

$$V^{\text{exit}}(s) = 1_{\{\text{continue} \mid \text{exit}\}}(s) n + 1_{\{\text{ch11} \mid \text{exit}\}}(s) n_{11}^{\text{exit}}$$

- Liquidate if $n < 0$ and $n_{11}^{\text{exit}} > 0$ not feasible; restructure if $n < 0$ and $n_{11} > 0$ feasible
- Price of debt conditional on exit is

$$\begin{aligned} \tilde{Q}_{\text{exit}}(z, k', b') = & \mathbb{E}_{(s' \mid s)} \left[\wedge \left\{ 1_{\{\text{continue} \mid \text{exit}\}}(s') + 1_{\{\text{ch11} \mid \text{exit}\}}(s') \alpha_{11}^{\text{exit}} \right\} \right] \\ & + \mathbb{E}_{(s' \mid s)} \left[\wedge 1_{\{\text{ch7} \mid \text{exit}\}}(s') R(\omega', b', k') \right] \end{aligned}$$

Entry

- Mass $\bar{\mu}$ enter each period replacing exiting firms (for all reasons)
- Enter with capital $k = k_0$, $b = 0$ and $z \sim \Omega^e(z)$

Liquidation Choice: More General Setup

Long-term debt

- assume portion debt m matures each period (randomly) and nonmatured pays coupon c
- cash-on-hand: $n = \pi + q\omega(1 - \delta)k - [m + (1 - m)c]b$
- external funds: $Q(.) [b' - (1 - m)b] - qk'$
- default threshold: if $n \in [\underline{n}(z, b), 0)$ exposed to rollover crises and $n < \underline{n}(z, b)$ insolvent
- (recursive) debt prices (simplified = no bkruptcy, no exit, no discount, $c = 0$, $\alpha_7 = 0$):

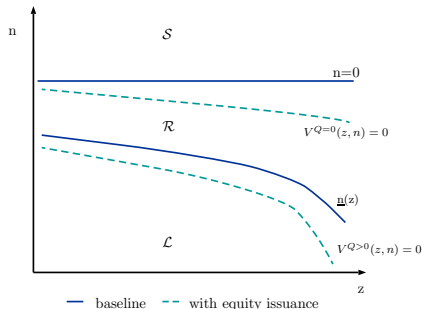
$$\tilde{Q}(z, k', b') = \mathbb{E}_{z'|z} \left[\left\{ 1_{n' \geq 0} + (1 - \eta) 1_{n' \in [\underline{n}(z', b'), 0)} \right\} \left\{ (1 - m) \tilde{Q}(z', k'', b'') + m \right\} \right]$$

More general (assume $c = 0$ for exposition)

- profits $\pi(z, k)$ gral z process, invest $\iota(k, k')$ allow for idio k frictions and long-term debt
- dividends (if no roll crises) are $d = \pi(z, k) - \iota(k, k') - bm + \tilde{Q}(.) (b' - (1 - m)b)$
- multiple eq if $\max_{k', b'} d \geq 0$ and $n \equiv \pi(z, k) - bm - \iota(k, 0) < 0$ hold

Liquidation Choice: Costly Equity Issuance

- Firms can issue equity $e < 0$ at cost $\phi(e)$, which is decreasing in e and unbounded.



- $\mathcal{S} : V^{Q=0}(z, n) \geq 0$
- $\mathcal{L} : V^{Q>0}(z, n) < 0$
- $\mathcal{R} : V^{Q>0}(z, n) \geq 0$ and $V^{Q=0}(z, n) < 0$

- Where $V^{Q=0}$ firm problem with costly equity issuance where $Q = 0$ and $V^{Q>0}$ same but with $Q > 0$

Rollover Risk

- Firm-level credit spread

$$\frac{\tilde{r}(z, b', k') - r}{1 + \tilde{r}(z, b', k')} = \underbrace{\Pr \left[(n', z') \in \mathcal{L} \mid z \right]}_{\text{insolvency risk}} + \underbrace{\eta \Pr \left[(n', z') \in \mathcal{R} \mid z \right]}_{\text{rollover risk}}$$

assume no rollover crises today, no restructuring, $\Lambda = 1/(1+r)$, $\alpha_7 = 0$ and $\gamma = 0$

- Aggregate-level incidence of rollover crises

$$\underbrace{\int_{z, n < \underline{n}(z)} d\Omega(z, n)}_{\text{insolvent firms}} + \eta \times \underbrace{\int_{z, n \in [\underline{n}(z), 0)} d\Omega(z, n)}_{\text{exposed firms}}$$

note: if $\eta \rightarrow 0$ then back to model with defaults only driven by fundamentals

Bankruptcy and Debt Prices

- Firm-level credit spreads ($\Lambda = 1/(1+r), \alpha_7 = 0, \gamma = 0$) [full Q](#)

$$\frac{\tilde{r} - r}{1 + \tilde{r}} = \underbrace{\mathbb{E} \left[1'_{\{\text{ch7}\}} \right]}_{\text{liquidation}} + \underbrace{\mathbb{E} \left[1'_{\{\text{ch11}\}} \times (1 - \alpha'_{11}) \right]}_{\text{restructuring}}$$

- Model nests

► $c \rightarrow \infty$

$$\frac{\tilde{r} - r}{1 + \tilde{r}} \approx \mathbb{E} \left[1'_{\{\text{ch7}\}} \right] = \Pr \left[(n', z') \in \mathcal{L} \mid z \right] + \eta \Pr \left[(n', z') \in \mathcal{R} \mid z \right]$$

► $c = 0$ and $\alpha_{11} \rightarrow 1$ then spreads are

$$\frac{\tilde{r} - r}{1 + \tilde{r}} \approx \mathbb{E} \left[1'_{\{\text{ch7}\}} \right] = \Pr \left[(n', z') \in \mathcal{L} \mid z \right]$$

Corporate Debt Prices

- $Q = [1 - \mathbf{1}_{\text{Ch7}}(s)]\tilde{Q}$ from creditor's no profit condition
- \tilde{Q} determined by (discounted) $\mathbb{E}[\text{prob tomorrow's bankruptcy events}]$

$$\begin{aligned}\tilde{Q}(z, k', b') &= (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[\Lambda 1_{\{\text{continue}\}}(s') \times 1 \right] \\ &\quad + (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[\Lambda 1_{\{\text{Ch11}\}}(s') \times \alpha_{11}(s') \right] \\ &\quad + (1 - \gamma) \mathbb{E}_{(z'|z, \omega', \phi')} \left[\Lambda 1_{\{\text{Ch7}\}}(s') \times R(k', b', \omega') \right] \\ &\quad + \gamma \tilde{Q}_{\text{exit}}(z, k', b')\end{aligned}$$

where

- $\alpha_{11}(s)$ recovery rate of creditors if restructure bargain protocol
- $R(k, b, \omega) = \min \{1, \alpha_7 (1 - \delta) q \omega k / b\}$ recovery rate if liquidated
- $\tilde{Q}_{\text{exit}}(z, k', b')$ debt price conditional on exit shock Q with exogenous exit

Qualitative Identification of η

Proposition (Identification of η)

Assume that in the restructuring process the debt haircut is α_{11} fixed and bankruptcy costs are a fixed cost $c_{11} \in (0, -\underline{n}(z_{\max}))$ with z_{\max} highest productivity firm in the economy. Then for a given distribution of firms

- 1. if $\alpha_{11} \rightarrow 1$ then firms that are insolvent don't restructure their debt,*
- 2. if $\alpha_{11} \rightarrow 1$ then the share of firms that restructure their debt (i.e., $(z, n) \in \mathcal{R}$ with $n_{11} \geq n$) identifies η ,*
- 3. if $\alpha_7 < \alpha_{11} < 1$ then firms with higher debt require a smaller c_{11} to restructure.*

HH Problem

HH in equilibrium determines

$$\begin{aligned}\Lambda' &= \beta \frac{u_C(C', L')}{u_C(C, L)} \\ 1 &= E \left[\beta \frac{u_C(C', L')}{u_C(C, L)} (1 + r) \right] \\ w &= - \frac{u_L(C, L)}{u_C(C, L)}.\end{aligned}$$

with utility function $u_C(C, L) = \ln C - \Omega L$

Capital Producer

There is a representative aggregate capital producer that maximizes

$$\max_I q \Phi \left(\frac{I}{K} \right) - I$$

where I is the amount of final goods used to produce capital, K is the aggregate k stock, and $\Phi(.)$ is the aggregate capital adjustment cost function. FOC:

$$q = \frac{1}{\Phi' \left(\frac{I}{K} \right)}$$

- time-varying q and $\mathcal{R}(\cdot) \rightarrow$ financial accelerator mechanism (Bernanke, Gertler & Gilchrist 1999).

Steady-State Equilibrium

Steady-state equilibrium in this economy is Vfunctions of continuing firms $\{V, \tilde{V}\}$, decision rules $\{b', k', l\}$, aggregates $\{Y, C, I\}$, price schedule $Q(\cdot)$, interest rate r , prices $\{q, w\}$, default choices $1(\cdot)$, recov rates $\alpha_{11}(\cdot)$ and distribution of firms $\{\Omega(\cdot)\}$

- HHs choices are determined by Euler eq, SDF and labor supply eq [detail](#)
- price of capital q determine in K producer problem [detail](#)
- debt price satisfy no-profit condition of fin intermediaries [detail](#)
- given prices, firm's dec. rules solve the producing firm's problem [detail](#) and default choices are consistent with Default Propositions
- recovery rates satisfy bargaining protocol
- markets clear (labor, resources)
- distribution of firms **fixed point** in law of motion [detail](#)

Law of Motion States

Let Ω be the distribution of firms that produce which they a mass of 1, $\tilde{\Omega}$ the distribution of incumbent firms at the beginning of the period, g and \hat{g} the pdf of ω and ϕ respectively, p the conditional pdf of the productivity shocks ϵ_z , and Ω^e the distribution of entrant firms. To define the equilibrium first we need to determine the law of motion of the distribution. Distribution of firms that produce is

$$\begin{aligned}\Omega(z, n) = & (1 - \gamma) \int \left[1_{\{\text{ch11}\}}(s) 1_{\{n^{11}(z, k, b, \omega) = n\}} + 1_{\{\text{cont}\}}(s) 1_{\{n(z, k, b, \omega) = n\}} \right] d\tilde{\Omega}(s) \\ & + \bar{\mu}(1 - \gamma) \int \left[1_{\{\text{ch11}\}}(s) 1_{\{n^{11}(z, k_0, 0, \omega) = n\}} + 1_{\{\text{cont}\}}(s) 1_{\{n(z, k_0, 0, \omega) = n\}} \right] \hat{g}(\phi) g(\omega) d\phi d\omega d\Omega^e(z) \\ & + \text{lom} \mid \text{exit}\end{aligned}$$

The distribution of incumbent firms at the beginning of the period $\tilde{\Omega}(z, \omega, k, b, \phi)$ is

$$\tilde{\Omega}(s') = \int 1_{\{k'(z, n) = k'\}} 1_{\{b'(z, n) = b'\}} \hat{g}(\phi') g(\omega') p(\epsilon_z \mid \rho_z z + \epsilon_z = z') d\epsilon_z d\Omega(z, n)$$

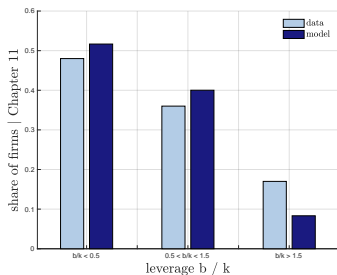
Data Sources, Sample and Some Definitions

Compustat

- Two samples (accounting changes after 2018, see Ma's online notes)
 - ▶ Pre-covid = 1980-2017 (n=179k annual, n=426 k quarterly)
 - ▶ Covid = 2019-2020 (n=14k quarterly)
- Sample selection: nonfinancial, $k > 0$, assets > 0 , drop outliers and short-spell (< 20 q spell)
- Key definitions:
 - ▶ n = profits + liq value capital – net liquid liabilities
 - ▶ profits = $F1.oiadpq$ where $F1$ = one period ahead in the data
 - ▶ net liquid liabilities = $1ctq - cheq$
 - ▶ liq value capital = $inv tq \times \omega_{inv} + rectq \times \omega_{rec} + ppentq \times \omega_{ppentq} + acoq$ where ω_x is liq value rate (from Kermani Ma 2020) of asset class x
- Identify bankrupt firms that operate following Corbae D'Erasmus 2021. Use footnote to total assets and deletion information ($d1rsn$ and $d1dte$). Bankrupt firms:
 1. report adoption accounting under Ch11, or bankrupt and not deleted
 2. data available next period

Untargeted Moments of Bankruptcy

Distribution of leverage b'/k'
firms in Chapter 11



Predictors of Chapter 11

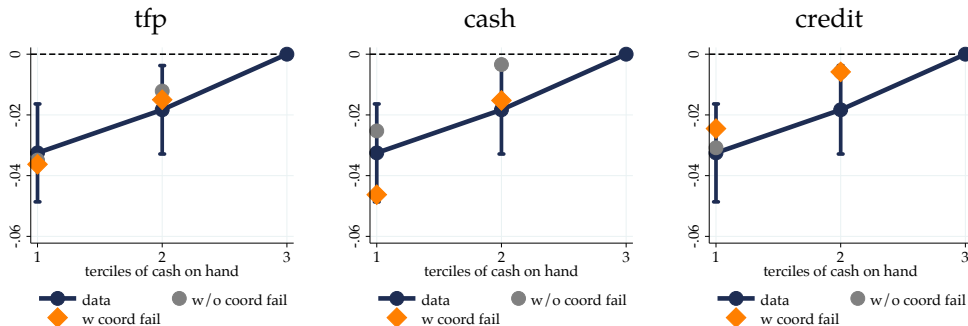
	dependent variable: $1_{i,t}^{ch11}$					
	(1)		(2)		(3)	
	data	model	data	model	data	model
$n_{i,t-1}/k_{i,t}$	-0.39 (0.03)	-0.05			-0.39 (0.10)	-0.45
$b_{i,t}/k_{i,t}$			0.11 (0.04)	0.03	-0.29 (0.09)	-0.41
$\log(k_{i,t-1})$	-0.50 (0.12)	-0.06	-0.52 (0.12)	-0.06	-0.49 (0.12)	-0.10
$d \log(\text{sales}_{i,t-1})$	-0.04 (0.00)	-0.03	-0.04 (0.00)	-0.02	-0.04 (0.00)	-0.01
Sector FE	Y		Y		Y	
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y		Y		Y	
Observations	370,973		373,362		370,973	

$$\text{empirical specification: } 1_{i,t}^{ch11} = \beta X_{i,t-1} + \alpha_t + \alpha_i + \alpha_s + \epsilon_{i,t}$$

Investment Adjustment Heterogeneity

- Estimate heterogeneity in Δk adjustments during crises [empirical specification](#) [measurement](#)
- Data and model simulation for Great Recession and Covid [episodes](#)

Heterogeneity of $\Delta k(\text{crisis}) - \Delta k(\text{no crisis})$

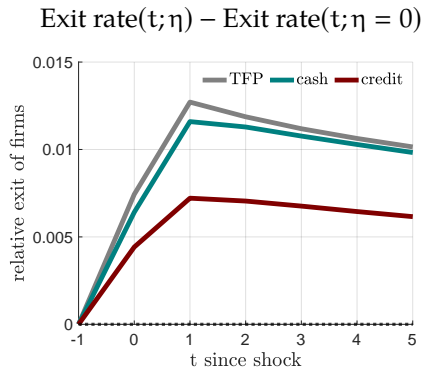
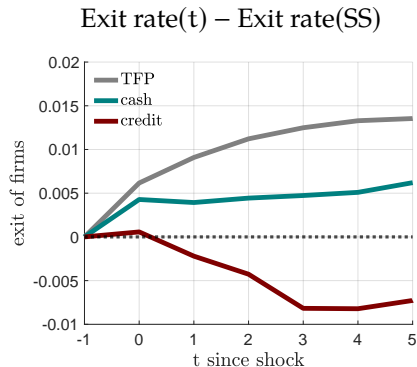


note: simple average of both episodes [individual episode](#) [empirical results](#) [other shocks](#)

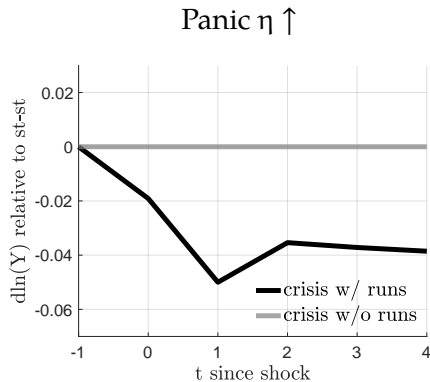
[back](#)

Firm (net) Exit During Recessions

- Firm exit dynamics during crisis experiments



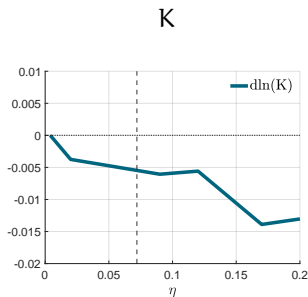
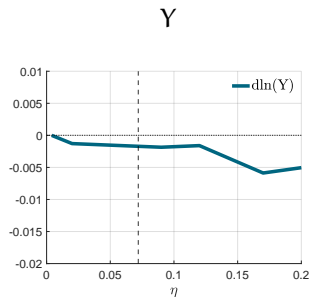
Rollover Crises and Panics



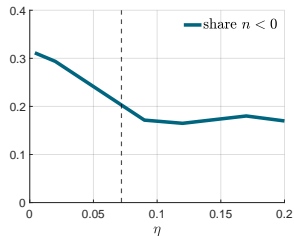
[back](#)

Steady State Comparison

- Variables: aggregate income Y , capital, K and share of firms with $n < 0$
- Comparison: steady state for different η



share of firms with $n < 0$



notes: log difference relative to st-st with $\eta = 0$ for Y and K and levels for share of firms

[back](#)

Heterogeneous Investment Response

Empirical Specification

- Diff-in-diff crisis event estimate
similar to Kalemli-Özcan Laeven Moreno 2020

$$\Delta \log(k_{it}) = \underbrace{\sum_{j=1}^J \beta_j^n \left(Q_{it}^{nj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } n/k} + \underbrace{\sum_{j=1}^J \beta_j^b \left(Q_{it}^{bj} \times \text{crisis}_t \right)}_{\text{heterogeneity across } b/k} + \underbrace{\Lambda' Z_{it}}_{\text{controls}} + \varepsilon_{it}$$

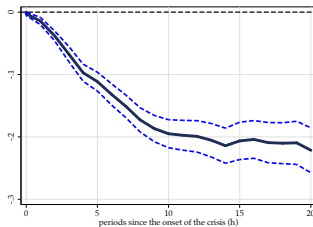
let $x_{it} = \{b_{it}, l_{it}\}$ firm i at period t with

- demeaned by sector $\hat{x}_{it} = x_{it} - \mathbb{E}_s[x_{it}]$.
- $\Delta \log(k_{it}) = \log(k_{it+h}) - \log(k_{it})$ with h peak-to-trough length
- crisis_t indicates if a crisis happens during the period considered
- $Z_{i,t}$: sales growth, log firm size, firm FE, sector FE

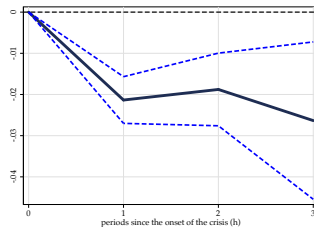
Recent Crisis Episodes in U.S.

Δ Capital Accumulation

(a) Great Recession



(b) Covid-19 Crisis



$$\beta_h : \log(k_{it+h}) - \log(k_{it}) = \alpha_i + \beta_h \text{crisis}_t + \varepsilon_{it+h}$$

[back](#)

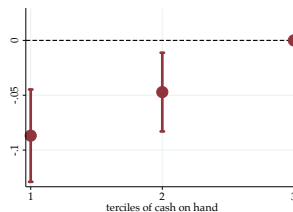
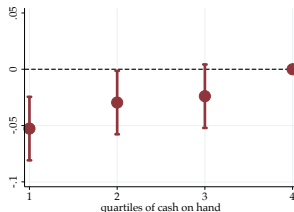
Investment Adjustment Heterogeneity

Recent Crisis Episodes in U.S.

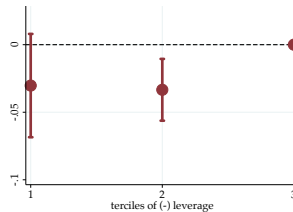
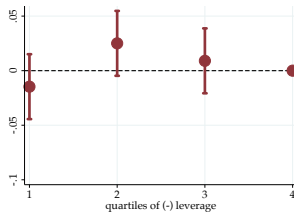
Great Recession

Covid-19 Crisis

Across n/k



Across b/k



Credit Policy Setup

- Announced unexpectedly at $t = 0$ (same period of shocks) for $T \geq 0$ periods and implemented at $j \in [0, T]$
- Eligible firms $(z, n) \in \mathcal{P}$ offer sequence of $\{Q_t^g(.)\}$
- Policy \mathcal{P} and labor taxes τ fixed across time
- Budget constraint from $t \geq 1$

$$\tau w_t L_t + B_t + B_{t-1,t}^g = B_t^g + (1 + r_{t-1}) B_{t-1}$$

B_t^g amount lent, $B_{t-1,t}^g$ lent at $t - 1$ and recovered at t

[back](#)

Credit Insurance Policy: First Best and Trade-off

Proposition (Credit Insurance Policy)

Assume that the government implements the credit insurance policy next period and is predictable today:

- 1. First best policy: $Q^g = \tilde{Q}$ then no rollover risk and qualified firms indifferent between using public or private credit.*
- 2. No screening: fix z^g such that $Q^g = \tilde{Q}(z^g, k', b')$ with firms qualified for credit those with $0 > n > \underline{n}(z^g)$. This policy faces a trade-off between lowering firm rollover risk and greater misallocation.*

- 1st best policy eliminates rollover crises and firms don't use the program's credit
- W/o screening greater z^g will preclude more rollover crises, but firms with $z < z^g$ will draw funds (zombification)

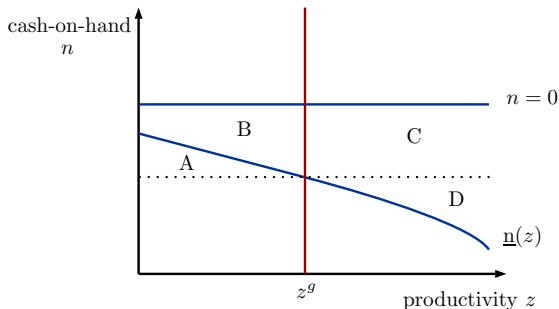
Direct Lending vs. Credit Guarantees

- Examples: direct lending \approx Fed's PMCCF SMCCF and credit guarantee \approx PPP
- In the theory policies are
 - ▶ direct lending (DL): alternative $Q^g(.)$ [detail theory](#)
 - ▶ credit guarantee (CG): repay $\alpha_g^r \geq \alpha^r$ in case of default
- Workings relative to rollover crises
 - ▶ DL affects payoffs (outside eq) and could coord creditors in good eq
 - ▶ CG relaxes $\underline{n}(z)$ but doesn't *directly* preclude rollover crises

Credit Policy Trade-off

Stylized example of 1 period policy in PE and two extreme cases

1. *Perfect screen of z* : $Q^g = \tilde{Q}$ and then remove coord failures for "free"
2. *No screen of z* : gov lends to eligible firms $n \in (0, \underline{n}(z^g)]$ firms at $\tilde{Q}(z^g, k', b')$, with z^g parameterizing policy scope



$A \cup B \cup C$: eligible

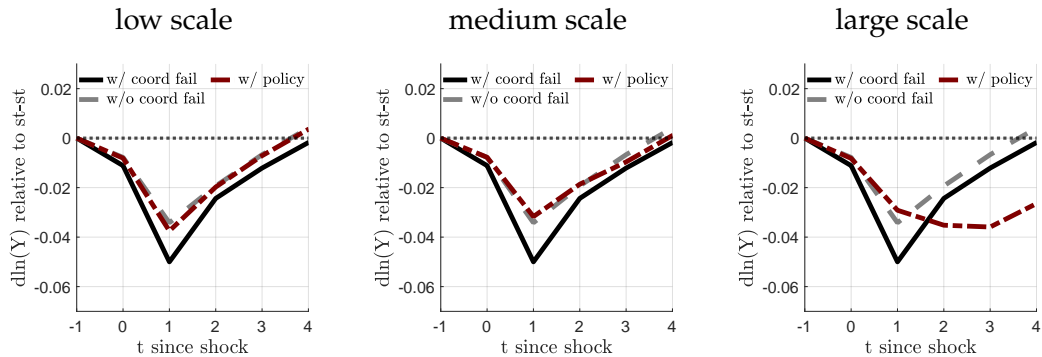
D: excluded

$A \cup B$: subsidized credit

$B \cup C$: rollover crises precluded

Credit Policy Implications: cash shock

- Policy active for first two periods and TFP shock driven crisis [back cash shock results](#)

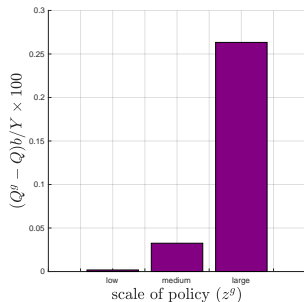


Credit Policy Implications: TFP shock

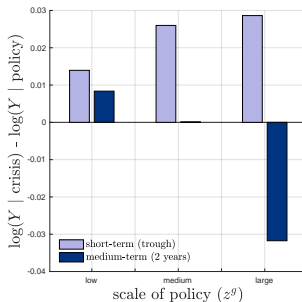
- Compute fiscal costs, short and long term benefits [back](#)

Costs and benefits

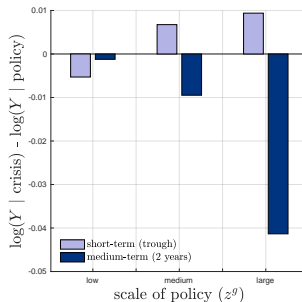
Fiscal costs



Benefits with roll crises



Benefits without roll crises



Liability Structure Data

- Debt maturity (Compustat)

	Time to mature (share)		
	< 1 year	1 to 4 years	≥ 5 years
Debt	0.29	0.33	0.38

	< 1 year	> 1 years
	0.61	0.39
Liabilities		

- Number of creditors from bankruptcy filings to Chapter 11 (FJC-IDB)

	# Creditors		
	1 to 100	101 to 1,000	>1,000
Medium (> 50 million and < 1 billion assets)	0.16	0.19	0.65
Large (> 1 billion assets)	0.03	0.04	0.93

How Costly are Firms' Rollover Crises?

- (ex-ante) Cost computed as $\tilde{Q}(z, k', b'; \eta) - \tilde{Q}(z, k', b'; 0)$
- Only 2.2% of the firms face a cost of rollover risk higher than intermediation spread

Cost of rollover crises (in annual spread terms) distribution

